AC⊕DC search: the winning solution to the Flywire VNC matching challenge

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Flywire Connectome

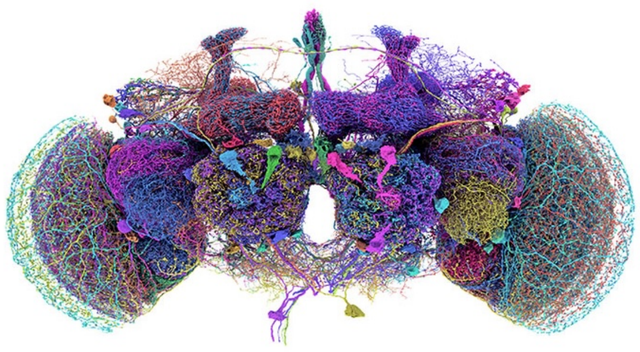


Image by Tyler Sloan and Amy Sterling flywire.ai

Ventral Nerve Cord Matching Challenge

Male Connectome Graph A B Female Connectome Graph

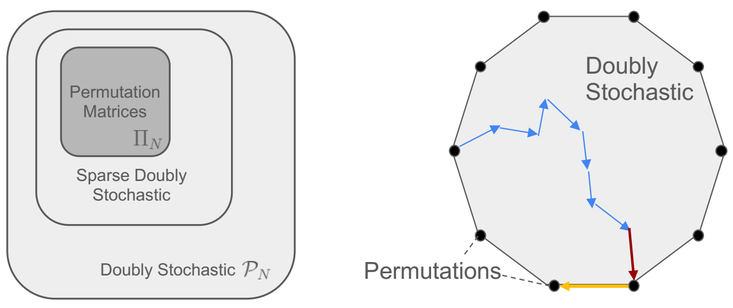
A_{ij} = # synapses from node i to node j in A

B_{ij} = # synapses from node i to node j in B

Let $\pi : \{1, 2, \ldots, n\} \to \{1, 2, \ldots, n\}$ be a one-to-one map from nodes in A to nodes in B.

Alignment score: $J(\pi) = \sum_{ik} \min\left(A_{ik}, B_{\pi(i)\pi(k)}\right)$

*A*lternating *C*ontinuous ⊕ *D*iscrete *C*ombinatorial **Search**



Frank-Wolfe updates using gradient of doubly stochastic matrix:

$$J(P) = \sum_{ijk\ell} P_{ij} P_{k\ell} \, \min(A_{ik}, B_{j\ell})$$

$$[\nabla J]_{ij} = \sum_{k\ell} \left[\min(A_{ik}, B_{j\ell}) + \min(A_{ki}, B_{\ell j}) \right] P_{k\ell}$$

Pairwise swaps over discrete permutations:

$S_{ij}(\pi)$ = effect on alignment score of swapping indices $(i \leftrightarrow j)$ prior to the action of permutation π

Leaderboard

Submitted	Name	Alignment Score
2025-02-18	Old School	5,853,925
2025-02-17	David A. Bader, Harinarayan Asoori Sriram, Srijith Chinthalapudi and Zhihui Du	5,853,910
2025-01-31	Old School Winner	5,853,779
2025-01-31	David A. Bader, Harinarayan Asoori Sriram, Srijith Chinthalapudi and Zhihui Du	5,849,534
2025-01-31	Yuxin Ma, Xiaowei Zhu and Liwang Zhu	5,842,347
2025-01-31	Wayne B Hayes, Marcus Longo and Russell Longo	5,841,041
2025-01-31	Team FAQ	5,838,188

Algorithms

procedure $\Delta = \textsc{EvaluateSwaps}(A, B, \pi)$

▷ *Compute gradient at π and permute rows and columns of B*

for $i \leftarrow 1$ to n **do**

for $j \leftarrow 1$ to n **do**

$G_{ij} \leftarrow \sum_{k=1}^{n} [\min(A_{ki}, B_{\pi_k j}) + \min(A_{ik}, B_{j\pi_k})]$

$B^{\pi}_{ij} \leftarrow B_{\pi_i \pi_j}$

▷ *Evaluate difference in scores due to pairwise swaps*

for $i \leftarrow 1$ to n **do**

for $j \leftarrow 1$ to n **do**

$H_{ii} \leftarrow \min(A_{ii}, B^{\pi}_{ii}) + \min(A_{ii}, B^{\pi}_{jj}) - \min(A_{ii}, B^{\pi}_{ij}) - \min(A_{ii}, B^{\pi}_{ji})$

$H_{jj} \leftarrow \min(A_{jj}, B^{\pi}_{ii}) + \min(A_{jj}, B^{\pi}_{jj}) - \min(A_{jj}, B^{\pi}_{ij}) - \min(A_{jj}, B^{\pi}_{ji})$

$H_{ij} \leftarrow \min(A_{ij}, B^{\pi}_{ii}) + \min(A_{ij}, B^{\pi}_{jj}) - \min(A_{ij}, B^{\pi}_{ij}) - \min(A_{ij}, B^{\pi}_{ji})$

$H_{ji} \leftarrow \min(A_{ji}, B^{\pi}_{ii}) + \min(A_{ji}, B^{\pi}_{jj}) - \min(H_{ji}, B^{\pi}_{ij}) - \min(A_{ji}, B^{\pi}_{ji})$

$\Delta_{ij} \leftarrow G_{i\pi_j} + G_{j\pi_i} - G_{i\pi_i} - G_{j\pi_j} + H_{ii} + H_{jj} - H_{ij} - H_{ji}$

procedure $(P_T, \Pi_T) = \textsc{DoFrankWolfe}(A, B, P_0, T)$

$P \leftarrow P_0$

for $t \leftarrow 1$ to T **do**

▷ *Compute gradient*

for $i \leftarrow 1$ to n **do**

for $j \leftarrow 1$ to n **do**

$G_{ij} \leftarrow \sum_{k\ell} \left[\min(A_{ki}, B_{\ell j}) + \min(A_{ik}, B_{j\ell}) \right] P_{k\ell}$

▷ *Project gradient, compute step size, and interpolate*

$Q \leftarrow \operatorname{argmax}_{Q \in \mathcal{P}} \text{trace}\left[G^{\top} Q\right]$

$\alpha \leftarrow \operatorname{argmax}_{\alpha \in [0,1]} \left[\mathcal{S}((1-\alpha)P + \alpha Q)\right]$

$P \leftarrow (1-\alpha)P + \alpha Q$

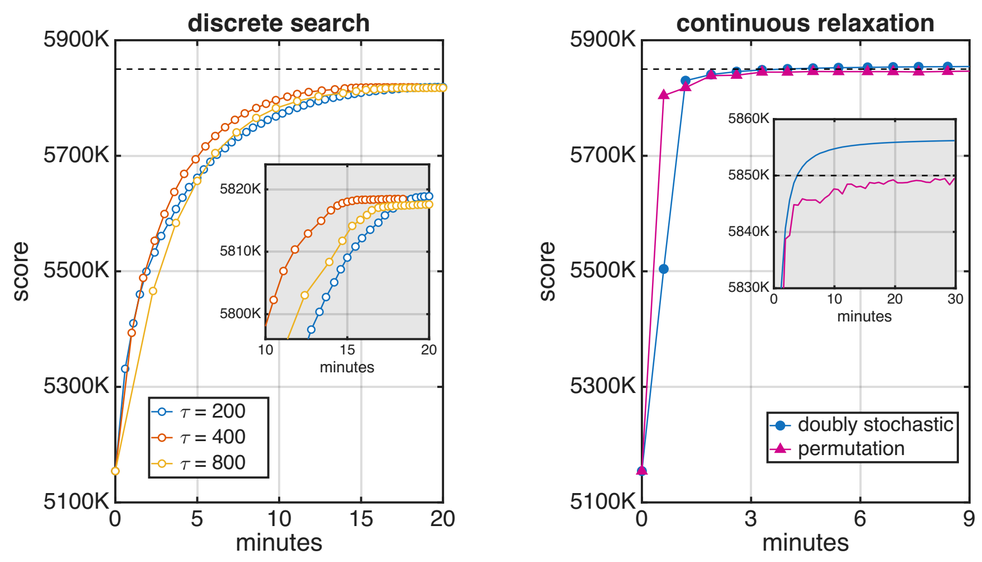
▷ *Compute closest-matching permutation matrix*

$\Pi \leftarrow \operatorname{argmax}_{\Pi \in \mathcal{P}} \text{trace}\left[P^{\top} \Pi\right]$

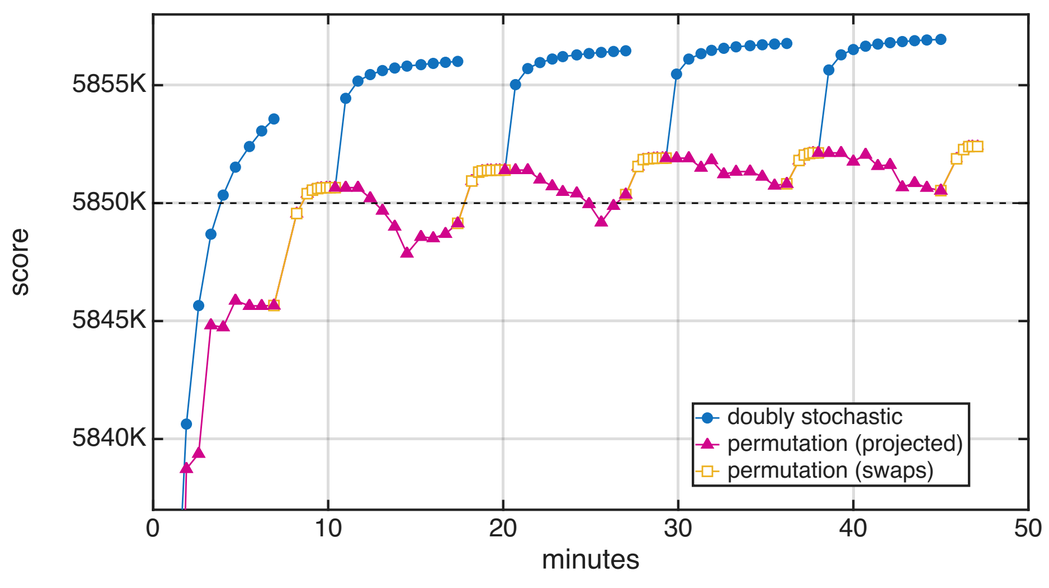
$P_T \leftarrow P$

$\Pi_T \leftarrow \Pi$

Discrete vs Continuous Search



AC⊕DC Search Results



NeurIPS Data on the Brain and Mind Workshop, December 2025, San Diego, CA