NeurIPS 2025 L2S

UCL

Learning Binary Sampling Patterns for Single-Pixel Imaging using Bilevel Optimisation

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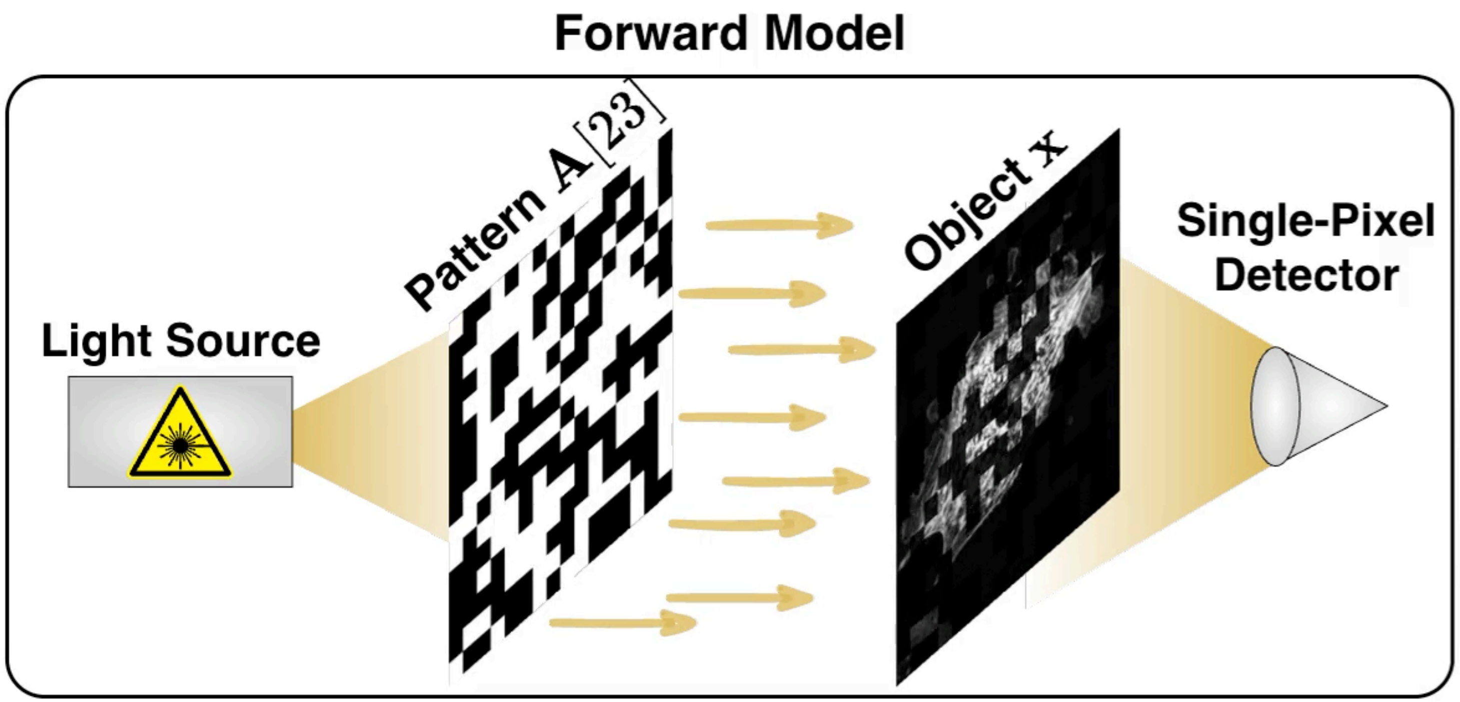
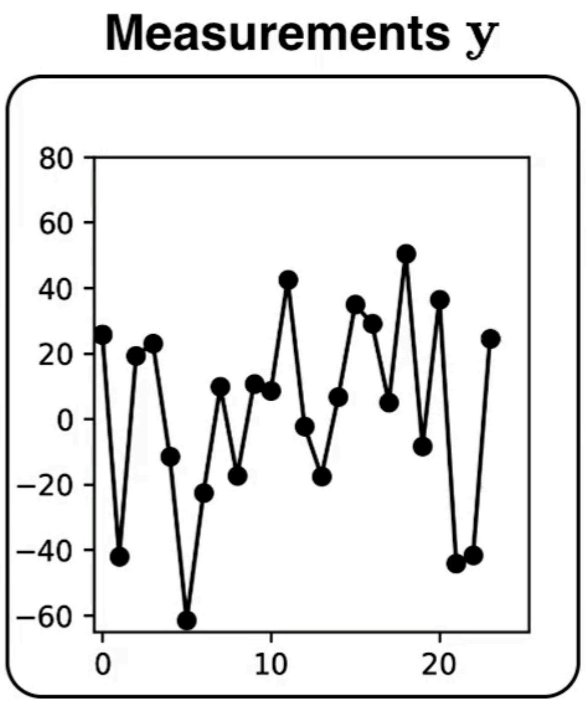
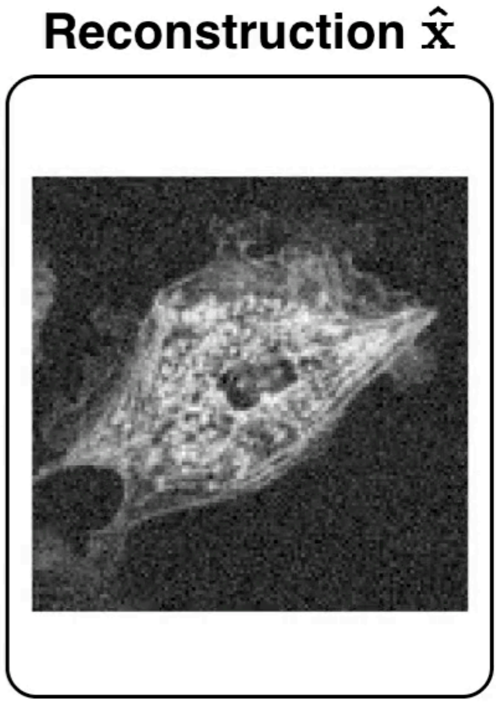
Image credits: Valerio Gandolfi - Politecnico di Milano

Single-Pixel Imaging







$$P(\mathbf{A}\mathbf{x}) = \mathbf{y}$$

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}\in\mathbb{R}^N} \frac{1}{2}\|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2^2 + \alpha\,\mathcal{J}(\mathbf{x})$$

COmputatioNal Imaging as a training network for Smart biomedical dEvices

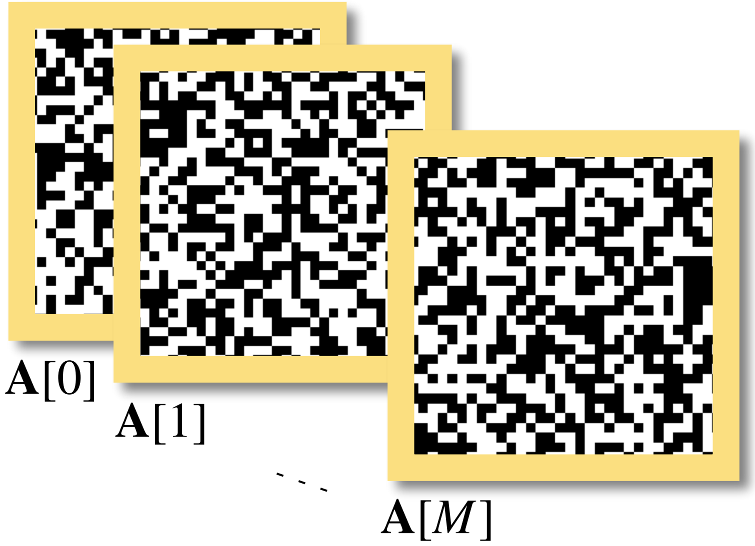
Goal

Design a sampling matrix $\mathbf{A}$ that:

- **Is binary** $\mathbf{A} \in \{-1, +1\}^{M \times N}$ (physical constraint)
- **Highly underdetermined** $M \ll N$ (application constraint)
- **Requires few training images** (application constraint)

Common Approaches

Acquisition: Scrambled Hadamard



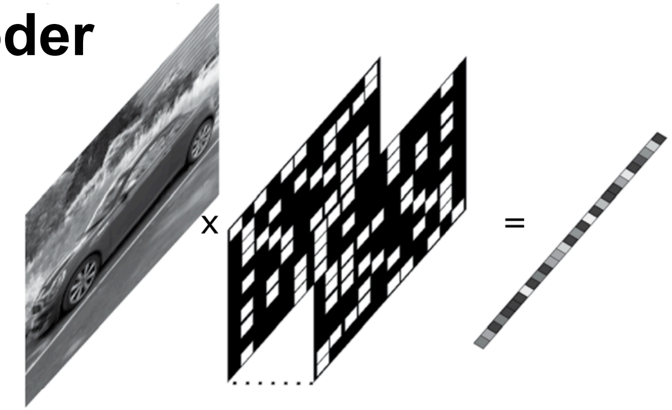
Reconstruction: Variational Regularisation

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}\in\mathbb{R}^N} \frac{1}{2}\|\mathbf{A}\mathbf{x}-\mathbf{y}\|_2^2 + \alpha\,\mathcal{J}(\mathbf{x})$$

with $\mathcal{J}(\mathbf{x}) = \mathrm{TV}(\mathbf{x}) = ||\nabla\mathbf{x}||_1$

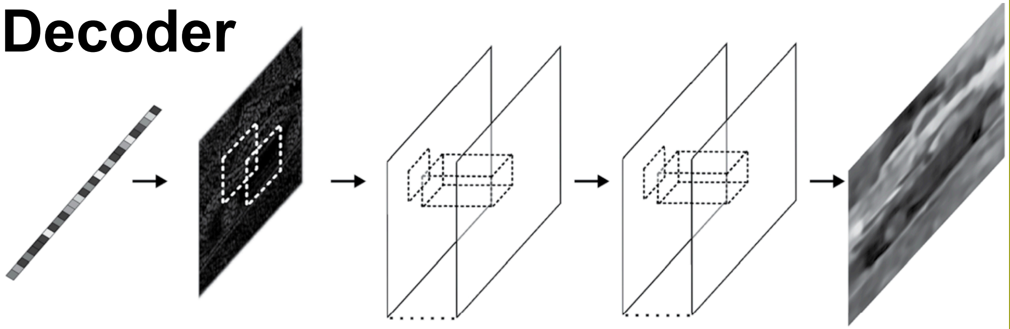
Acquisition: Deep Learning[1]

Encoder



Reconstruction: Deep Learning[1]

Decoder



[1] *Catherine F. Higham et al. "Deep Learning for Real-Time Single-Pixel Video" Scientific Reports 2018*

Common Approaches

Acquisition: Scrambled Hadamard

- 🟢 **Is binary $\mathbf{A} \in \{-1, +1\}^{M \times N}$**
- 🟡 **Highly underdetermined $M \ll N$**
- 🟢 **Requires few training images**

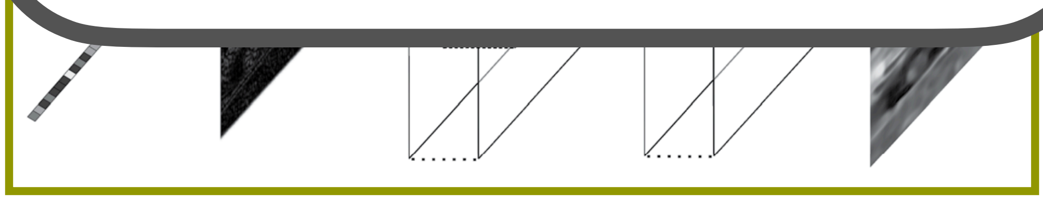
Rec ion

$\mathbf{x} \in \mathbb{R}$

with $\mathcal{J}(\mathbf{x}) = \mathrm{TV}(\mathbf{x}) = ||\nabla \mathbf{x}||_1$

Acquisition: Deep Learning[1]

- 🟡 **Is binary $\mathbf{A} \in \{-1, +1\}^{M \times N}$**
- 🟢 **Highly underdetermined $M \ll N$**
- 🔴 **Requires few training images**



[1] *Catherine F. Higham et al. "Deep Learning for Real-Time Single-Pixel Video" Scientific Reports 2018*

Our Contribution

We overcome current limitations with:

- Bilevel learning formulation:

$$\min_{\substack{\mathbf{A}\in\{-1,1\}^{M\times N}\\ \alpha>0}} \left\{ L(\theta) := \sum_{i=1}^{n} \mathcal{L}\Big(\mathbf{x}^{(i)}, \hat{\mathbf{x}}(\theta; P(\mathbf{A}\mathbf{x}^{(i)}))\Big) \right\}, \text{ where } \theta = (\mathbf{A}, \alpha),$$

$$\text{such that } \; \hat{\mathbf{x}}(\theta; \mathbf{y}) \in \arg\min_{\mathbf{x}\in\mathbb{R}^N} \frac{1}{2}\|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2^2 + \alpha\mathcal{J}(\mathbf{x}).$$

- Total Deep Variation[1] (TDV) as regulariser
- Surrogate gradient for binary constraint[2]

[1] Erich Kobler et al. “Total Deep Variation for Linear Inverse Problems”. In: Proceedings of the IEEE/CVF CVPR 2020.
[2] Yoshua Bengio et al. “Estimating or Propagating Gradients Through Stochastic Neurons for Conditional Computation” 2013.

Learning with Binary Constraints

Relax and Penalise[1]

- Relax the constraint to

$$\mathbf{A} \in [-1, 1]^{M \times N}$$

- Penalise with:

$$r_\epsilon(\mathbf{A}) = \frac{1}{\epsilon} \sum_{i,j} 1 - a_{i,j}^2$$

Straight-Through Estimator[2]

- Optimise over a latent matrix

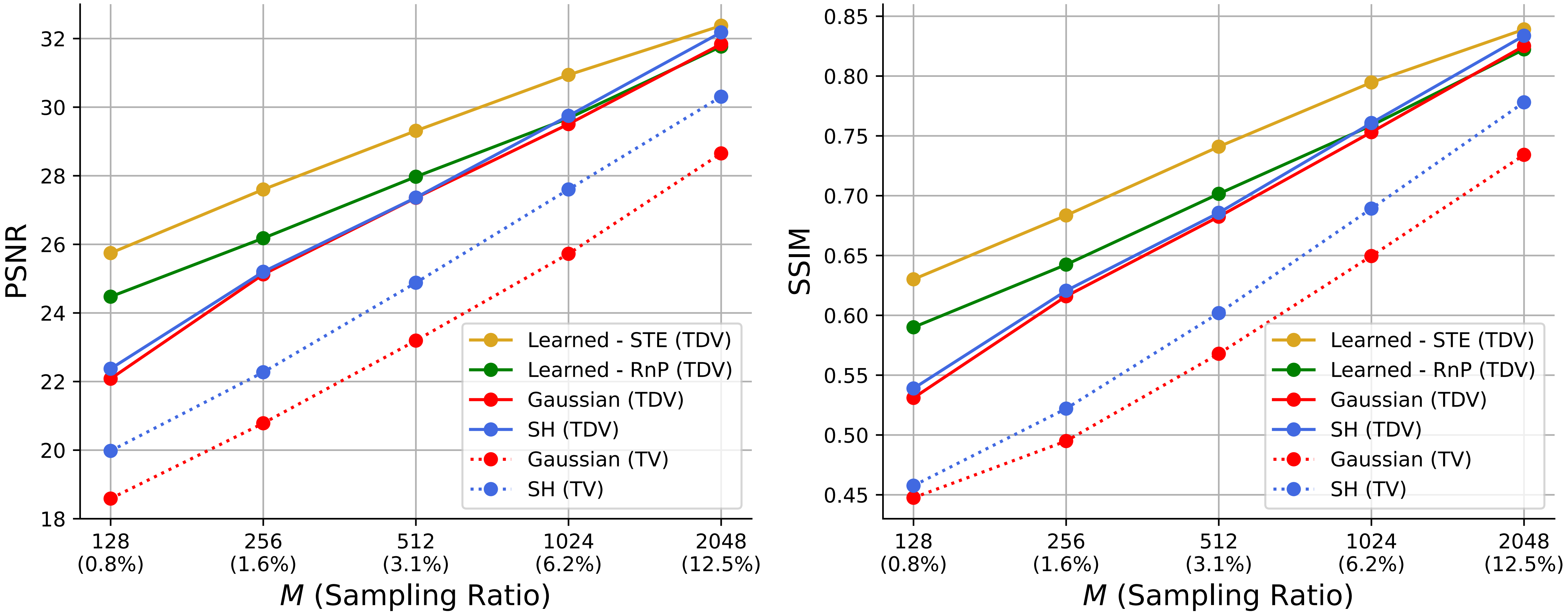
$\mathbf{Z} \in \mathbb{R}^{M \times N}$ where $\mathbf{A} = sgn(\mathbf{Z})$

- Use a surrogate gradient during backpropagation

$$tanh'(\mathbf{Z})$$

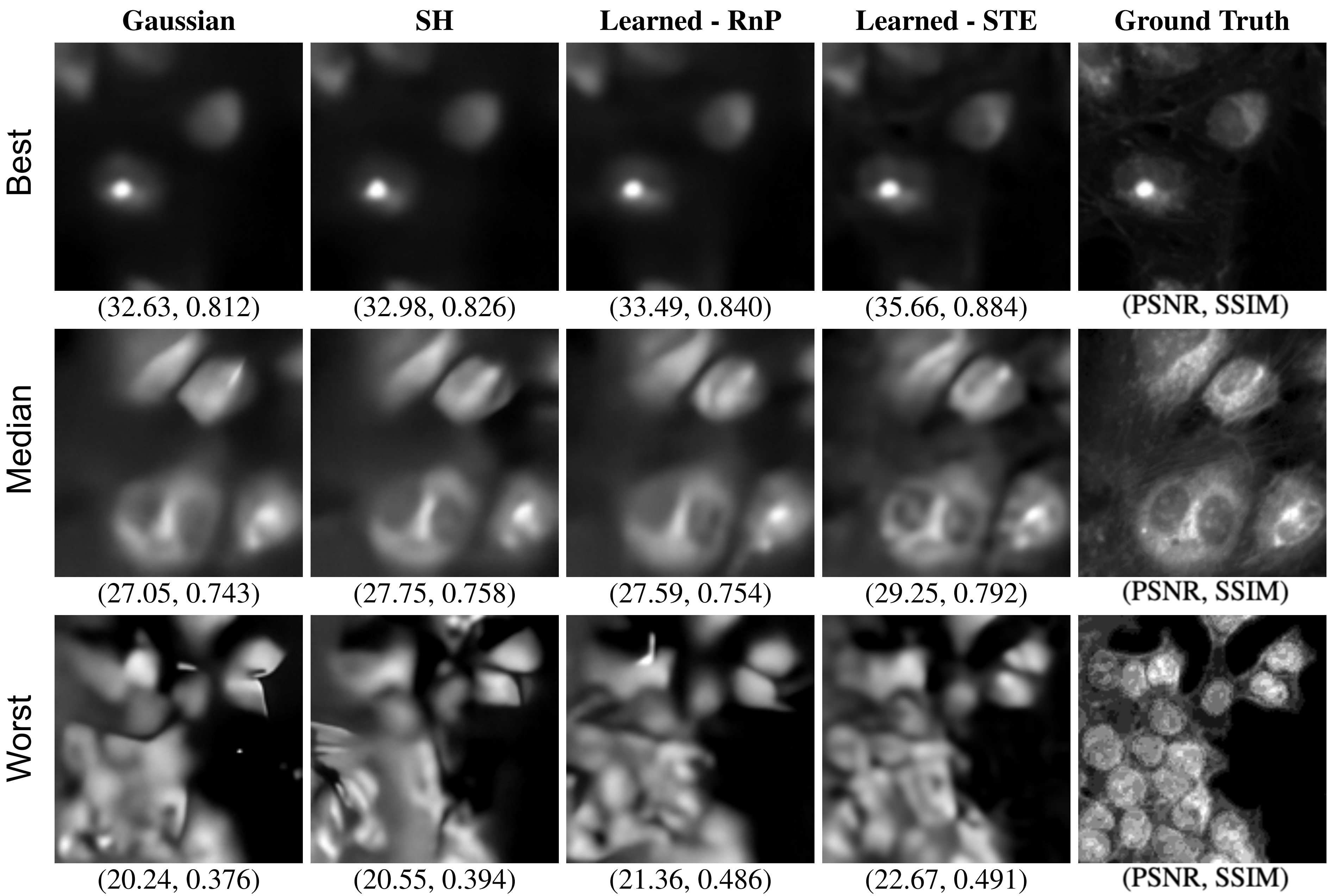
[1] S. Lucidi and F. Rinaldi. "Exact Penalty Functions for Nonlinear Integer Programming Problems". Journal of Optimization Theory and Applications 2010.
[2] Yoshua Bengio et al. "Estimating or Propagating Gradients Through Stochastic Neurons for Conditional Computation" 2013.

Results - Comparisons

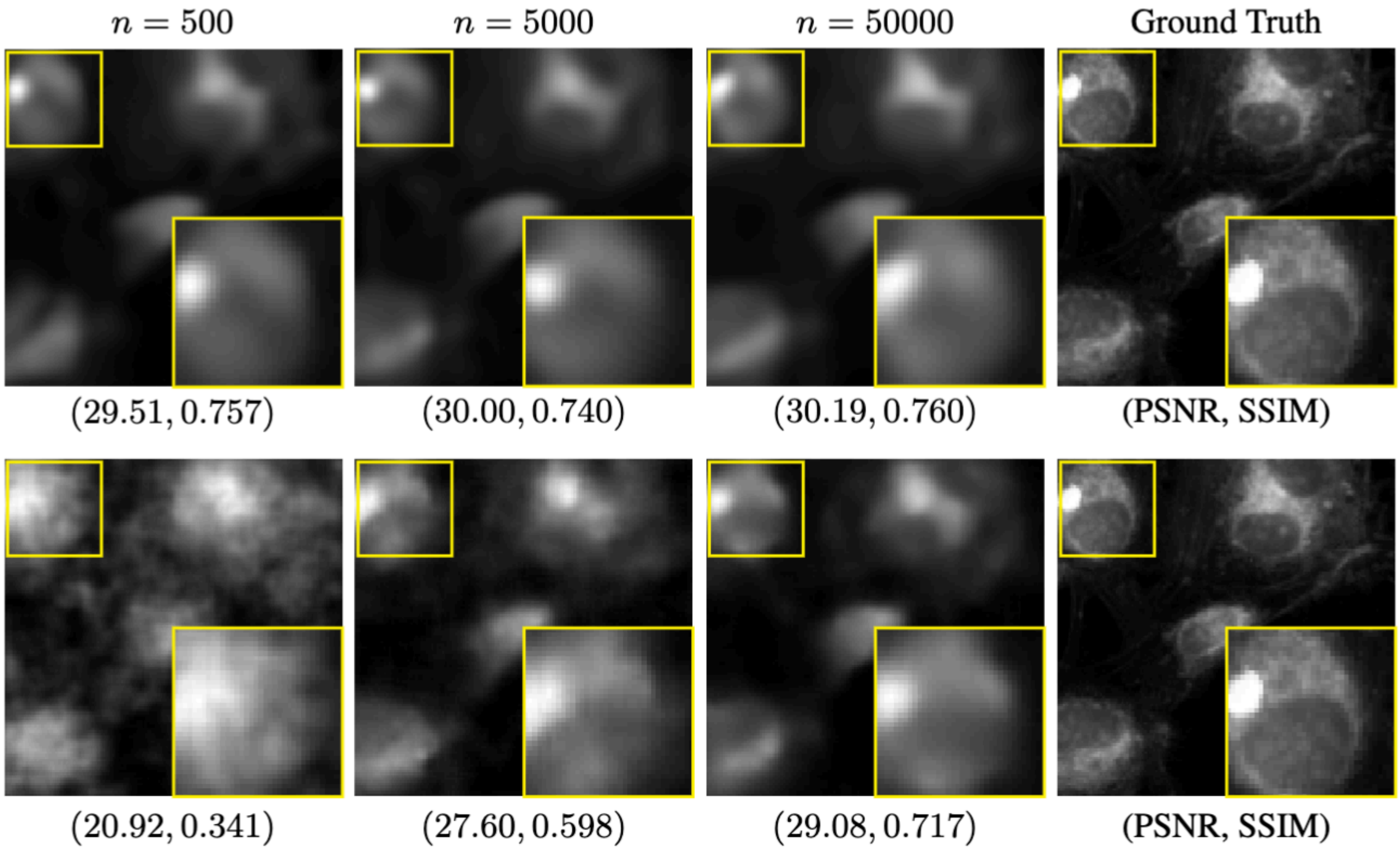


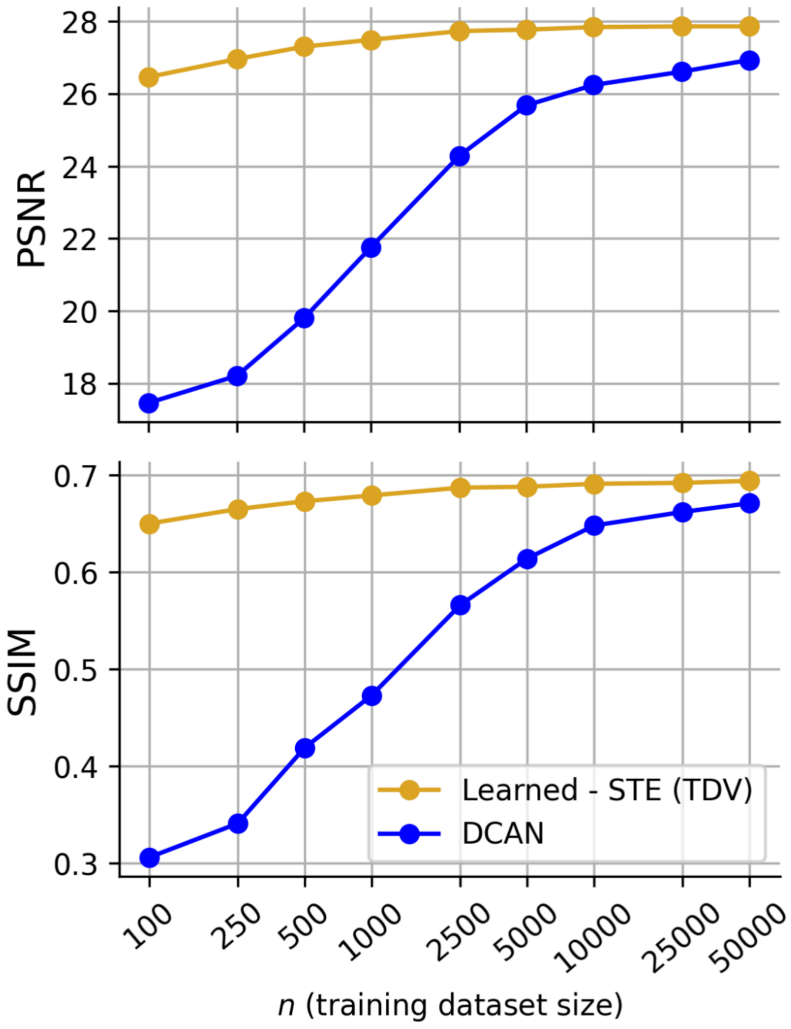
Results

Ordered by Learned - STE PSNR



Results - Comparison to E2E Deep Learning





Conclusions

✅ **Robust pattern design** for SPI

✅ High PSNR and SSIM in **data scarce settings**

📊 STE better **binarisation** choice to RnP

Future directions:

- Analysis of surrogate STE gradient
- Learned regularisers ablations (e.g. WCRR)
- Slope of surrogate gradient investigation

A massive thanks to the amazing people that made this possible:

Alexander Denker

Željko Kereta

Simon Arridge

CONcISE m4DL
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