Oak Ridge National Laboratory

Muslims in ML, NuerIPS 2025

A Quantum Machine Learning Algorithm for Solving Binary Constraint Problems

Presented by Sarah Chehade

Collaboration with Andrea Delgado and Elaine Wong

U.S. DEPARTMENT of ENERGY

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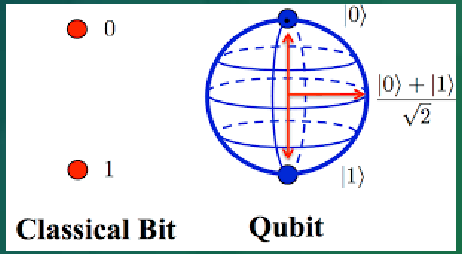
Quantum Computing 101

- **Model based on QUBITS instead of bits**
- $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle, \ |\alpha|^2 + |\beta|^2 = 1$



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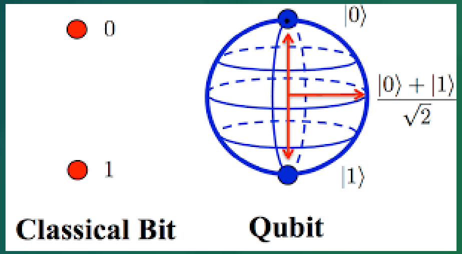

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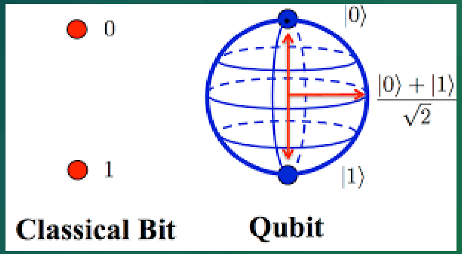

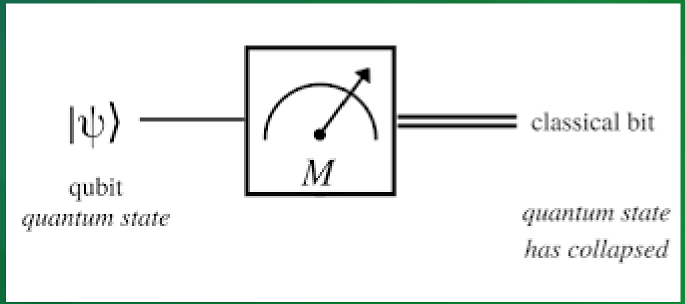

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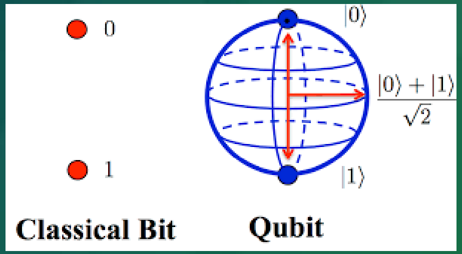
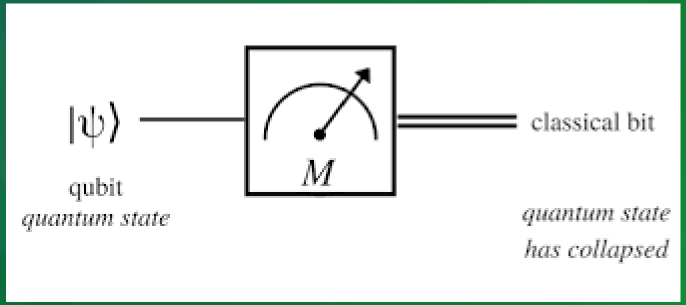
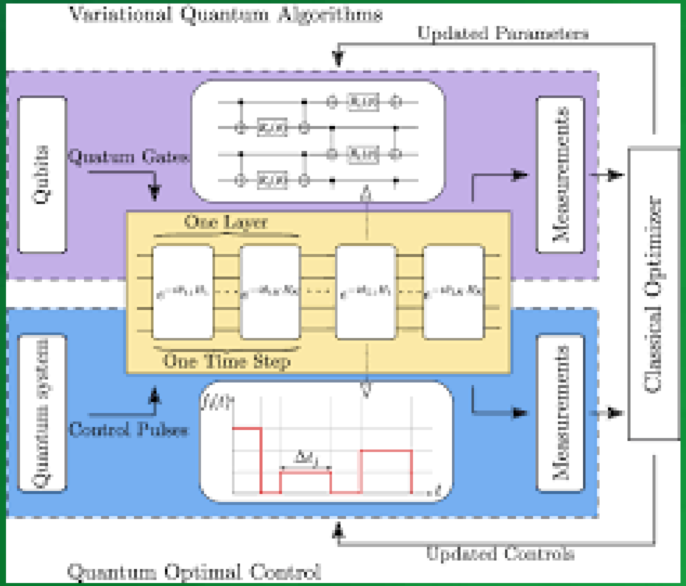
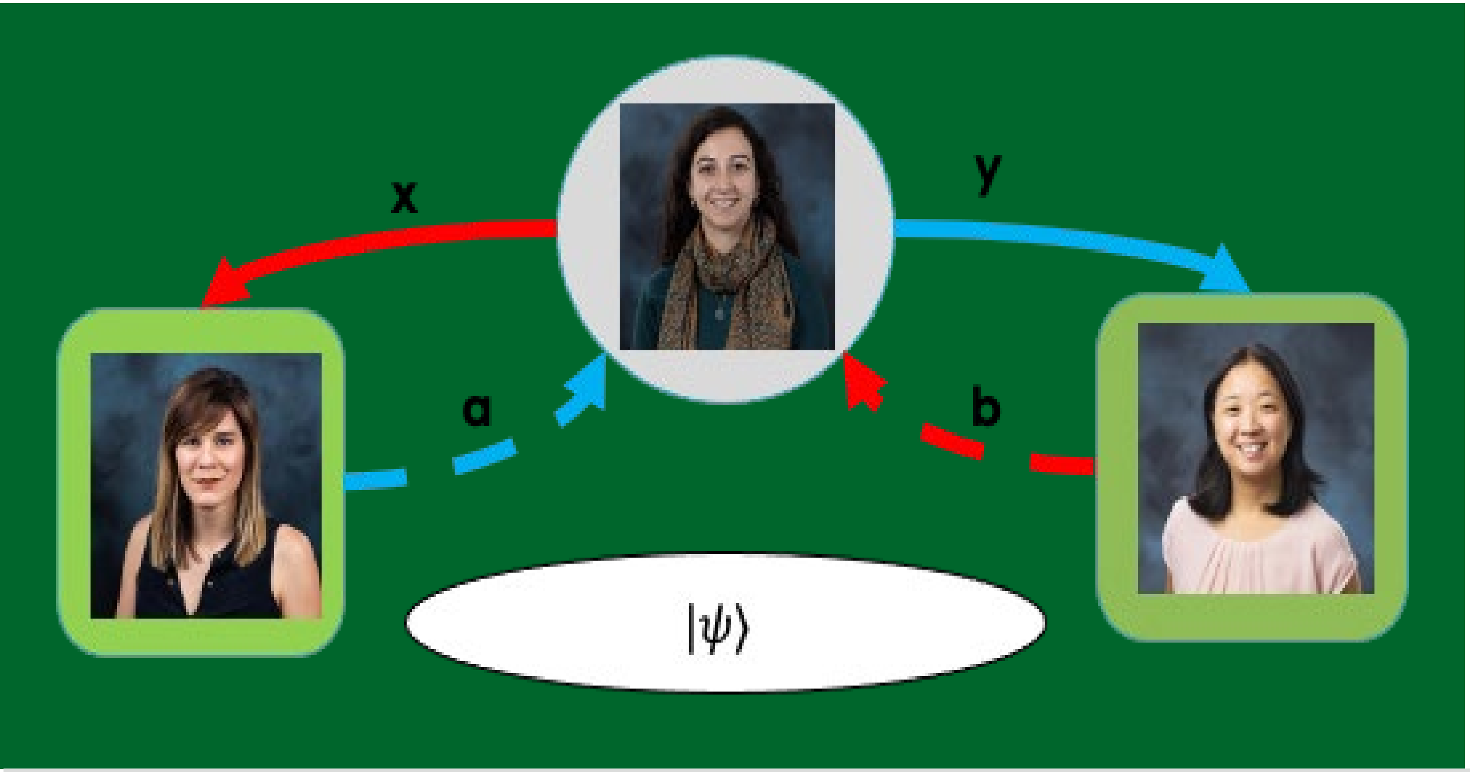
- Measurements: collapsing quantum data to classical data
- Computation = parameterized state construction + unitary operators + measurement
- (Mirrors deep learning: parameterized model, loss, optimization)







Non-local Games (NLGs): An Overview



$$val(G,\pi,p,\lambda) := \sum_{x\in I_A, y\in I_E} \pi(x,y) * \sum_{a\in O_A, b\in O_E} p(a,b|x,y) * \lambda(x,y,a,b)$$

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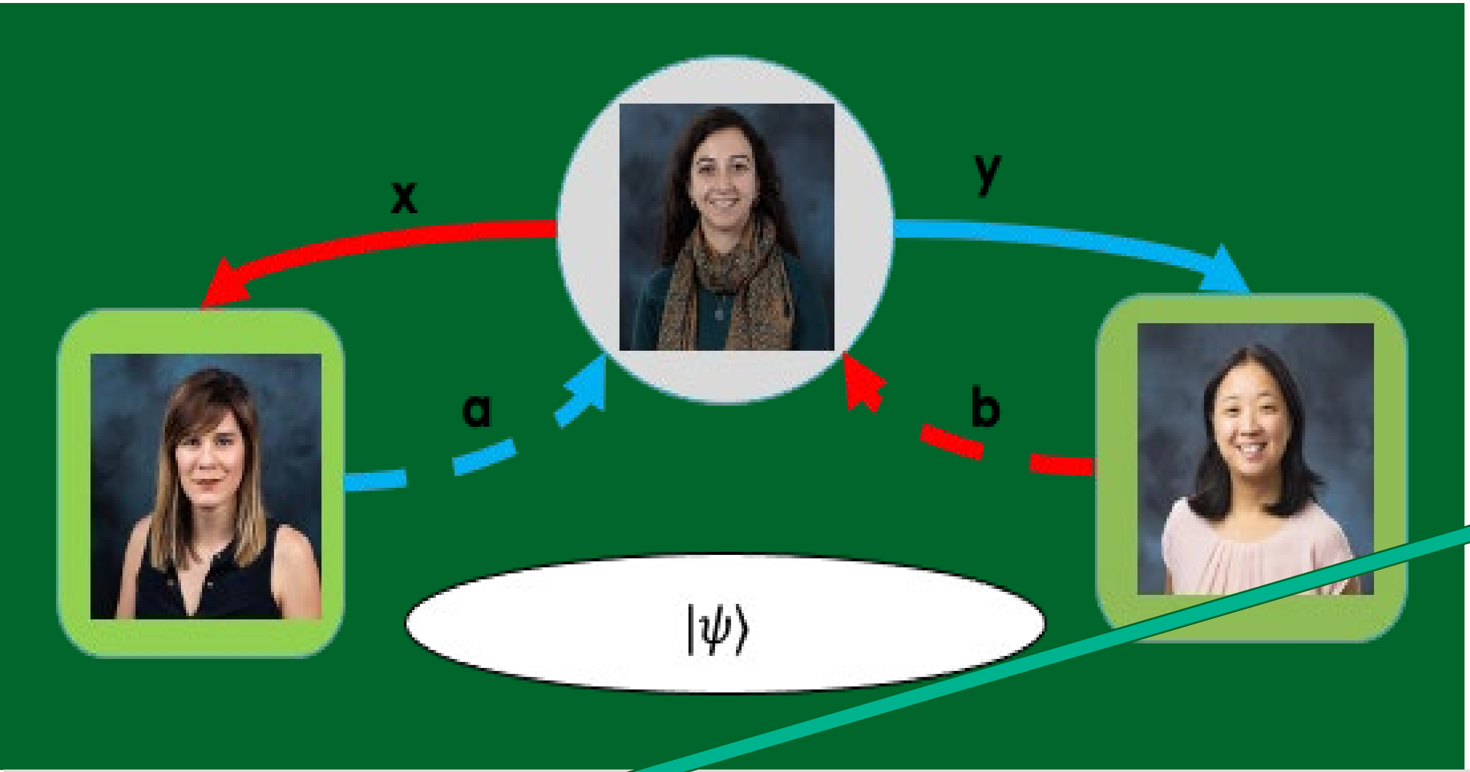

Computing is hard!

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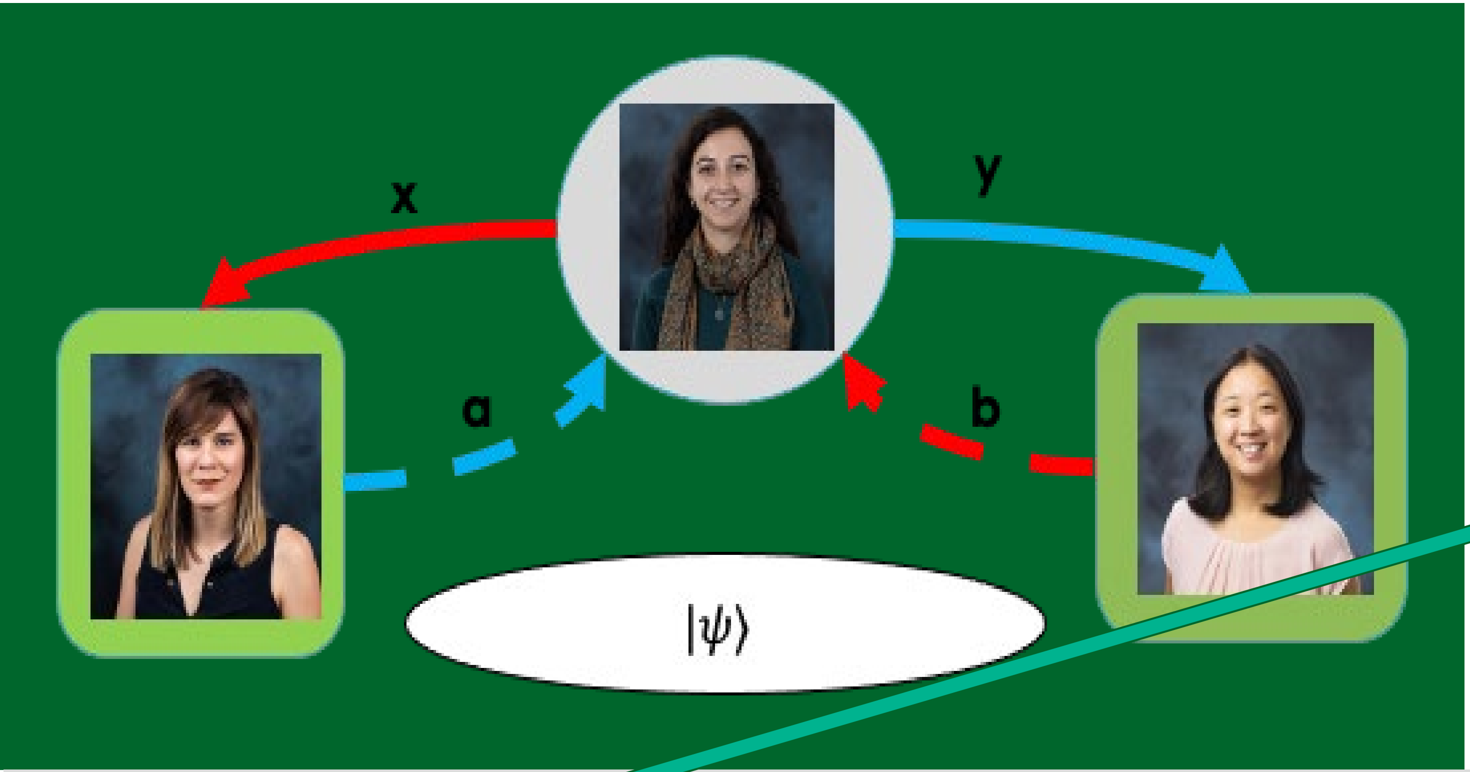


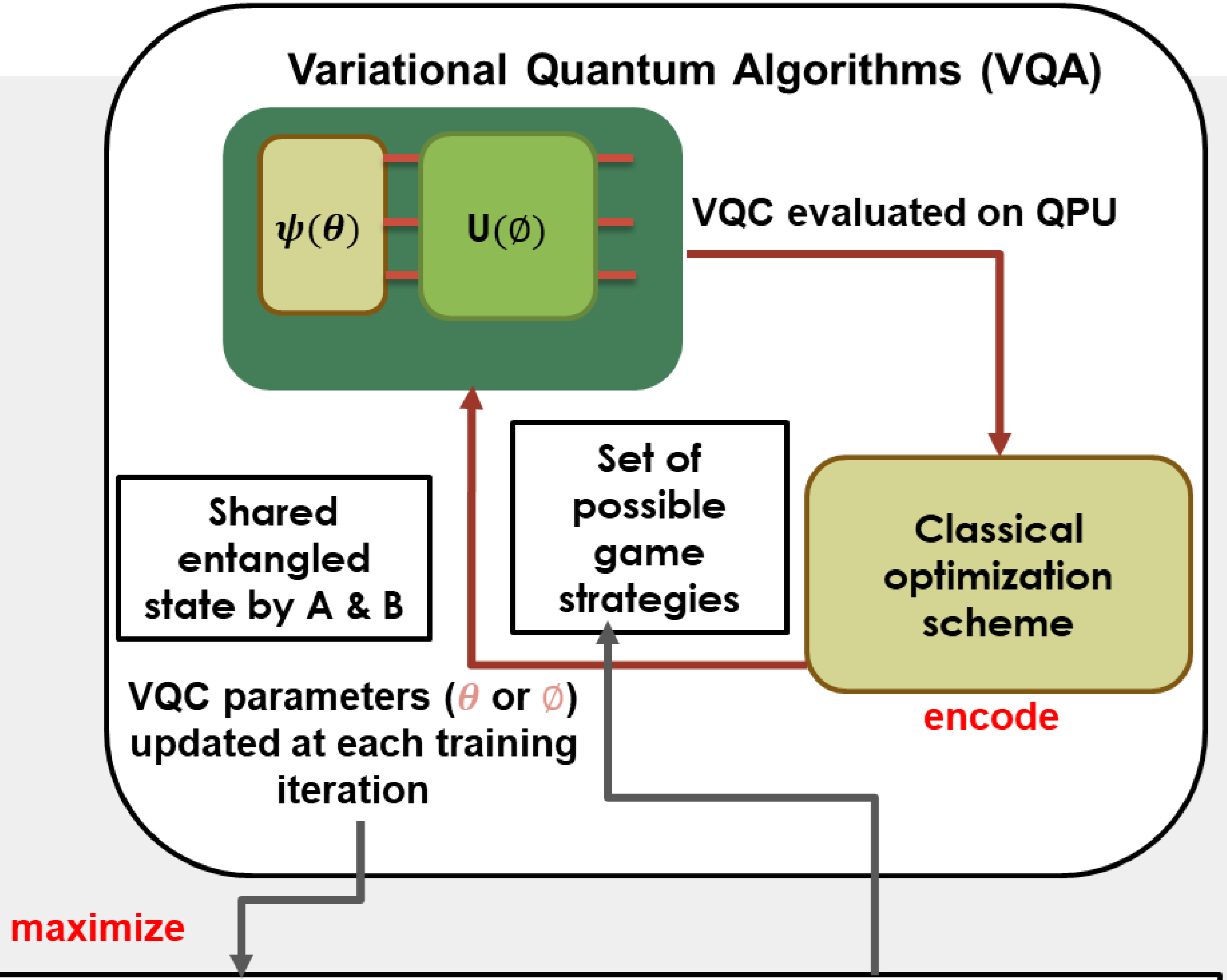
Computing is hard!

Lack of explicit strategies

$$val(G,\pi,p,\lambda) := \sum_{x\in I_A, y\in I_E} \pi(x,y) * \sum_{a\in O_A, b\in O_E} p(a,b|x,y) * \lambda(x,y,a,b)$$

Why Variational Quantum Algorithms (VQA)

Leverage the power of VQAs to compute game strategies and find the one that maximizes game value!



$$val(G,\pi,p,\lambda) := \sum_{x\epsilon\, I_A, y\epsilon\, I_E} \pi(x,y) * \sum_{a\epsilon\, O_A, b\epsilon\, O_E} p(a,b|x,y) * \lambda(x,y,a,b)$$

NLG example: Magic Squares Game (MSG)

Given a (3x3) grid, **Alice** is given 3 variables from a row while **Bob** is given 3 variables from a column. The goal is to assign 0 or 1 to each variable such that

$$v_1 + v_2 + v_3 = 0 \qquad v_1 + v_4 + v_7 = 0$$
$$v_4 + v_5 + v_6 = 0 \qquad v_2 + v_5 + v_8 = 0$$
$$v_7 + v_8 + v_9 = 0 \qquad v_3 + v_6 + v_9 = 1,$$

and Alice and Bob's variable assignments are consistent.

They cannot win perfectly with a classical strategy.

But they can win perfectly with a quantum strategy!

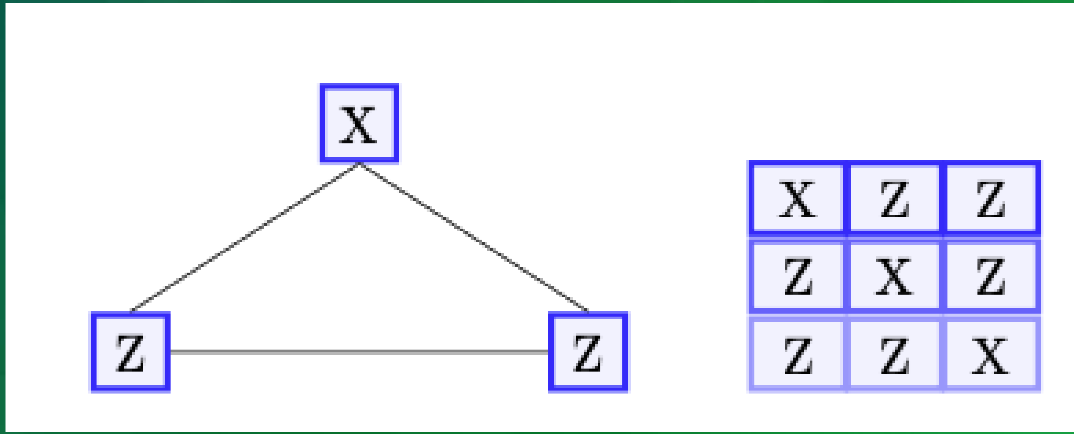
$Z \otimes 1$	$1 \otimes Z$	$Z \otimes Z$
$1 \otimes X$	$X \otimes 1$	$X \otimes X$
$Z \otimes X$	$X \otimes Z$	$Y \otimes Y$

v_1	v_2	v_3
v_4	v_5	v_6
v_7	v_8	v_9

v_1	v_2	v_3
v_4	v_5	v_6
v_7	v_8	v_9

How to construct the right game Hamiltonian

- **Pauli group construction:**
  hardware friendly
- **Parity guarantee:**
  1 Pauli X and 2 Pauli Z
- **Initial state:**
  3 Bell pairs: 6-qubit entangled state
- **Local commutativity:**
  allows simultaneous measuring
- **Measurement operators:**
  stabilizer logic



Algorithm Overview

- Construct a Hamiltonian that encodes the rules of the game

$$H = -\sum_{i,j} A_i \otimes B_j$$

- Initial state: $|\psi\rangle$
- Train `rotated` unitary measurement observables:

$$\tilde{A}_i = U_i^*(\theta) A_i U_i(\theta)$$

$$\tilde{B}_j = V_j^*(\phi) B_j V_j(\phi)$$

- **stronglyentanglinglayers**
- $L(\theta, \phi) = \langle\psi|(\sum_{i,j=0}^{2} \tilde{A}_i \otimes \tilde{B}_j)|\psi\rangle$

Row i **(Alice)**	**Column** j **(Bob)**	**Projector** $\Pi_{i,j}^{\text{win}} = \frac{1}{2}(\mathbb{I} + A_i \otimes B_j)$
$A_0 = Z \otimes Z \otimes X$	$B_0 = X \otimes Z \otimes Z$	$\frac{1}{2}(\mathbb{I} + A_0 \otimes B_0)$
$A_0 = Z \otimes Z \otimes X$	$B_1 = Z \otimes X \otimes Z$	$\frac{1}{2}(\mathbb{I} + A_0 \otimes B_1)$
$A_0 = Z \otimes Z \otimes X$	$B_2 = Z \otimes Z \otimes X$	$\frac{1}{2}(\mathbb{I} + A_0 \otimes B_2)$
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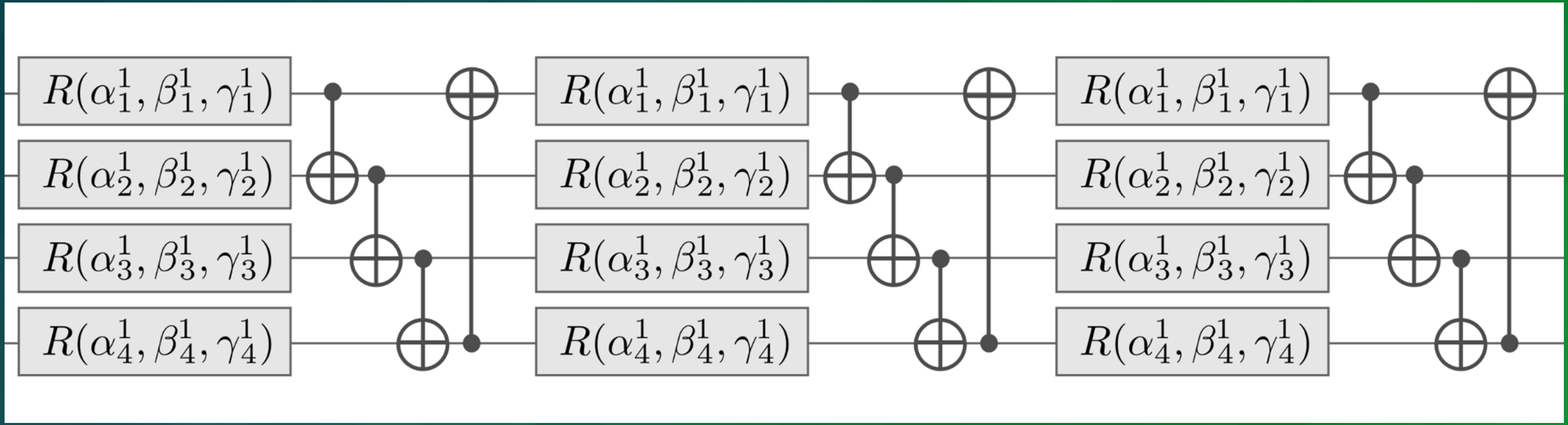
$A_2 = Z \otimes X \otimes Z$	$B_2 = Z \otimes Z \otimes X$	$\frac{1}{2}(\mathbb{I} + A_2 \otimes B_2)$

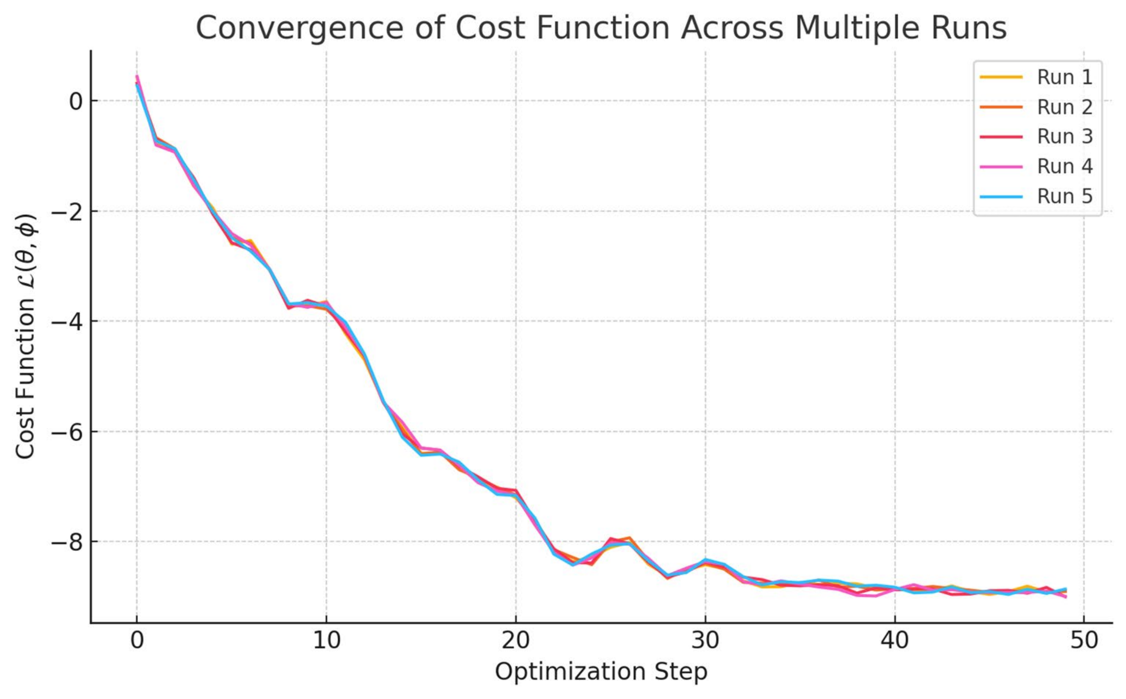
StronglyEntanglingLayers Circuit

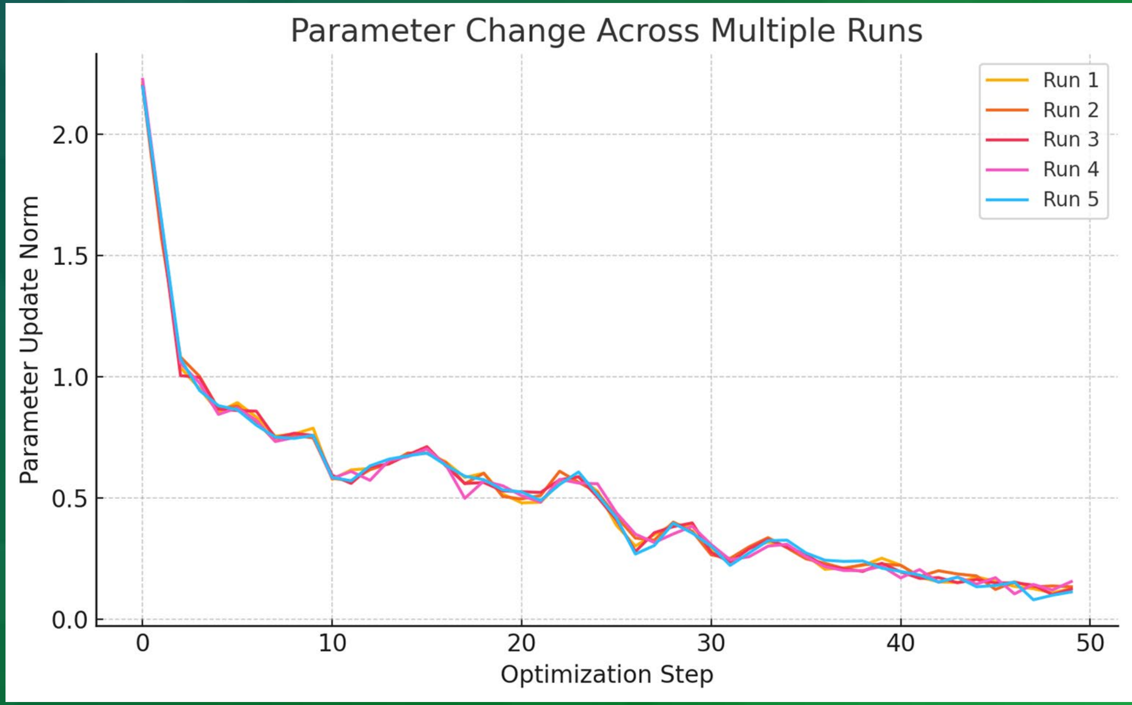
A built-in Pennylane function for variational quantum circuits.

Applies a sequence of parameterized single-qubit rotations followed by CNOT gates.

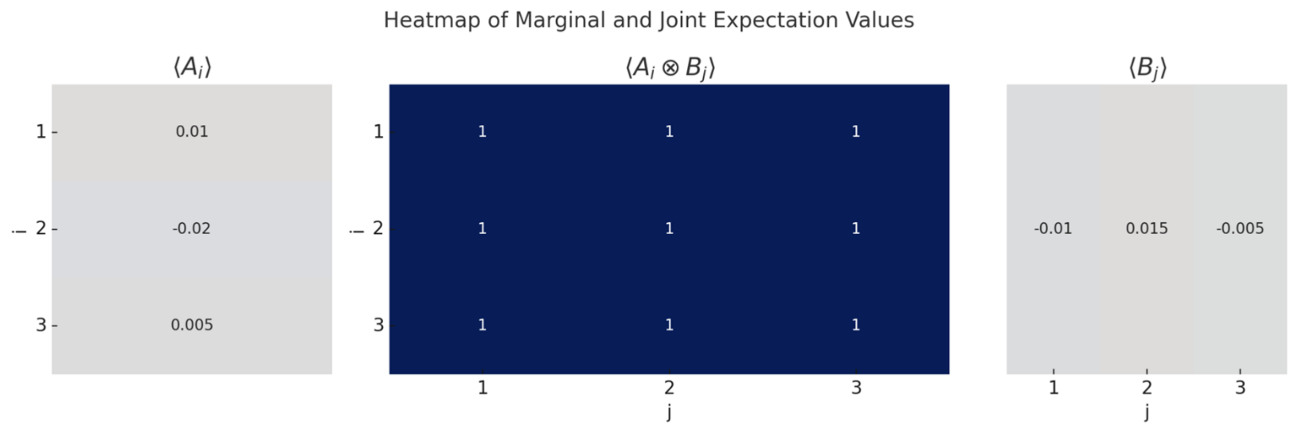


Results

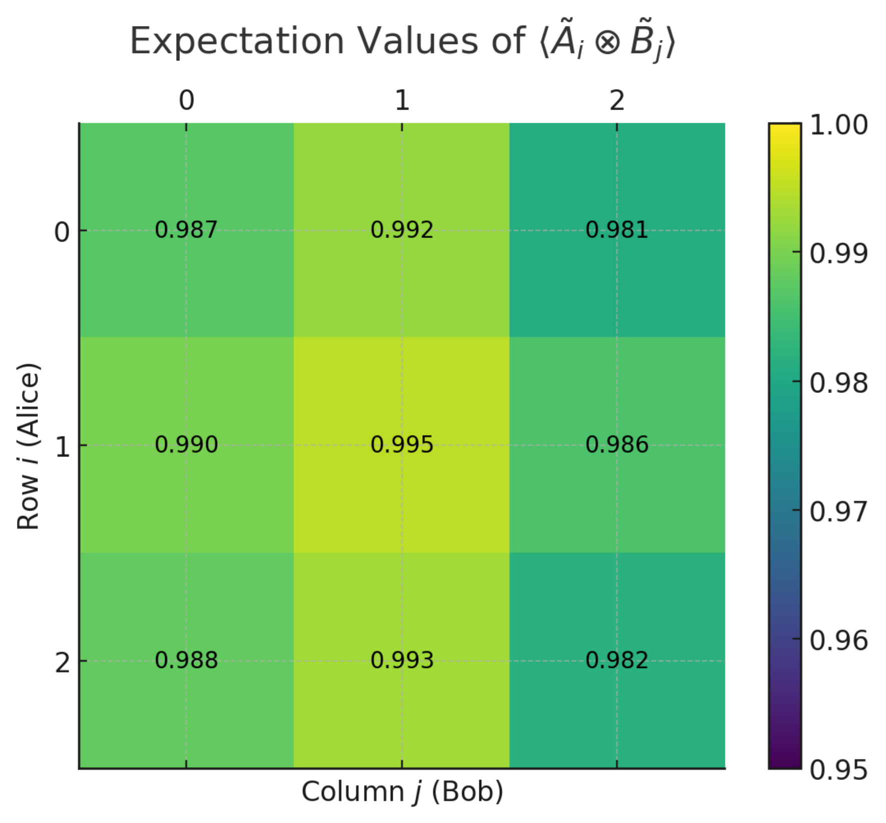

Convergence of Cost Function Across Multiple Runs
Run 1
Run 2
Run 3
Run 4
Run 5
Cost Function $\mathcal{L}(\theta, \phi)$
0
−2
−4
−6
−8
0
10
20
30
40
50
Optimization Step


Parameter Change Across Multiple Runs
Run 1
Run 2
Run 3
Run 4
Run 5
Parameter Update Norm
2.0
1.5
1.0
0.5
0.0
0
10
20
30
40
50
Optimization Step

Results



Heatmap of Marginal and Joint Expectation Values
⟨A_i⟩
⟨$A_i \otimes B_j$⟩
⟨B_j⟩
1
2
3
0.01
-0.02
0.005
1
1
1
1
1
1
1
1
1
-0.01
0.015
-0.005
j
i



Expectation Values of ⟨$\tilde{A}_i \otimes \tilde{B}_j$⟩
0
1
2
0.987
0.992
0.981
0.990
0.995
0.986
0.988
0.993
0.982
Row i (Alice)
Column j (Bob)
1.00
0.99
0.98
0.97
0.96
0.95

Acknowledgements

This work is supported by Oak Ridge National Laboratory's Laboratory Directed Research & Development (LDRD) Seed Program.

Chehade, Sarah, Andrea Delgado, and Elaine Wong. "A Game-Theoretic Quantum Algorithm for Solving Magic Squares." *preprint arXiv:2505.13366* (2025).

Thank you

Questions