Smooth Piecewise Cutting for Neural Operator to Handle Discontinuities and Sharp Transitions

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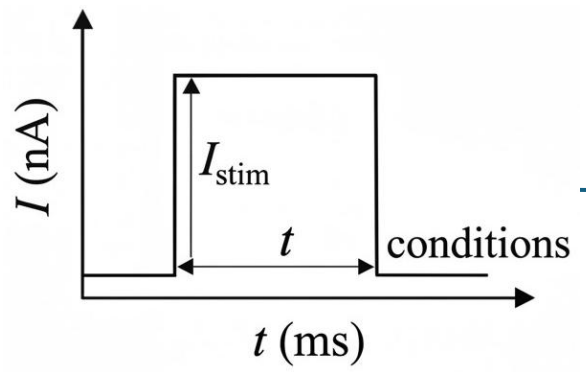
NEURAL INFORMATION PROCESSING SYSTEMS

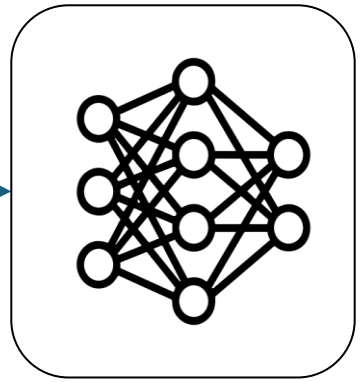
Neural Operator: A Mapping between Function Spaces

Traditional Solver: 1 Input → **Numerical Method/FEM** → 1 Output (Unpractical)

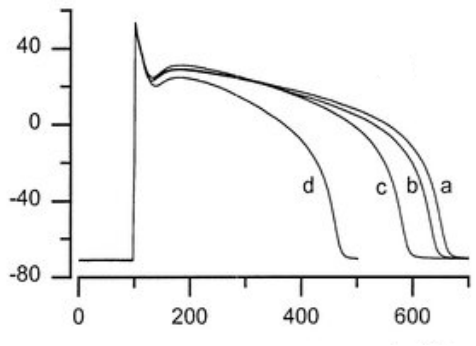
Neural Operator: n Input → **Network** → n Outputs (Fast solver)



Input, Domain
(Conditions, time, spatial, …)



Neural Operator
(DeepONet, FNO, …)

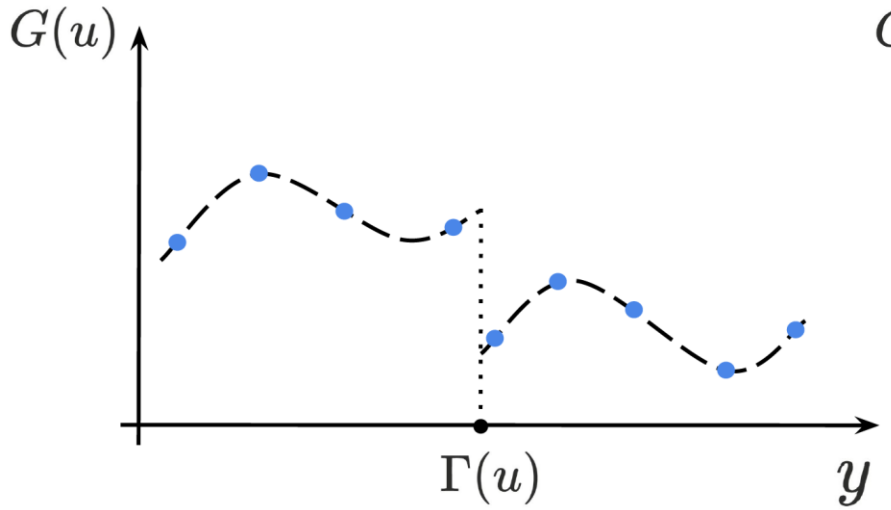


Solution Outputs

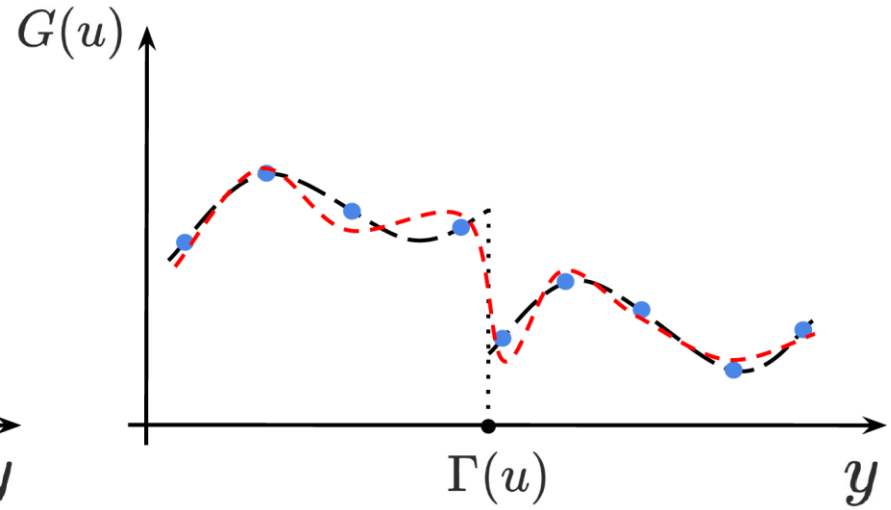
Benefit: Fast simulations can be used in full-scale optimal design or real-time optimal control

Limitation: A Continuous Representation for Discontinuous Function

Problem: Output solution have discontinuities/sharp transitions (hard to be approximated)



a) Ground truth (– – -) with discrete data (●)



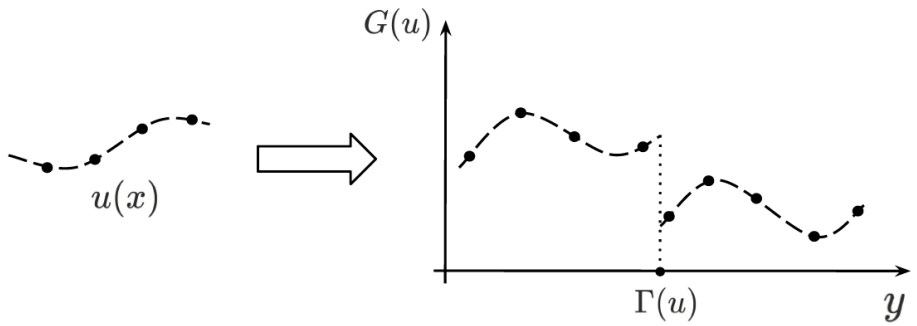
b) Continuous representation (– – -) based on data (●)

Neural operators inherit the continuity properties of neural networks, making it challenging to approximate discontinuous functions accurately.

⇒ **Accurate approximation of discontinuities often requires high-resolution data and high-frequency basis functions.**

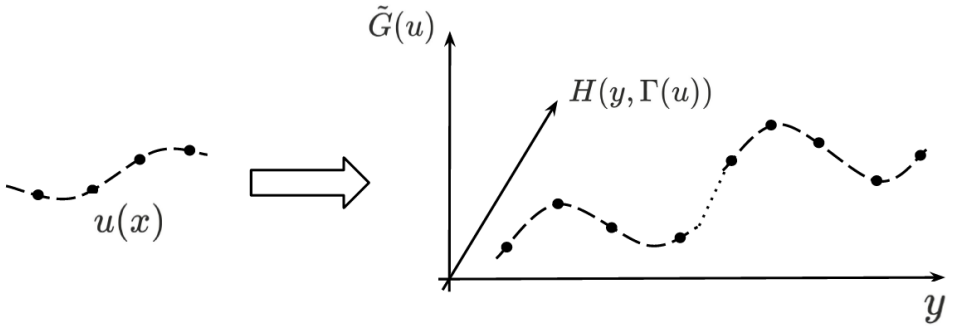
Cut-DeepONet (Our Approach)

Idea: A discontinuous function is a combination of several smooth parts.
⇒ Just focus on learning the smooth part, and ignore the discontinuity [1]



$\{(u_i, G(u_i))\}_{i=1}^N$ - Orignal Dataset

a) Output is discontinuous



$\{(u_i, \tilde{G}(u_i))\}_{i=1}^N$ - Lifted Dataset

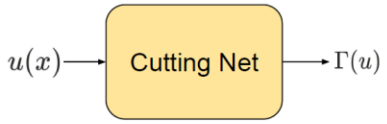
b) After lifting, output is continuous

[1] Ha Dang et al. “Smooth Piecewise Cutting for Neural Operator to Handle Discontinuities and Sharp Transitions”, *2026*

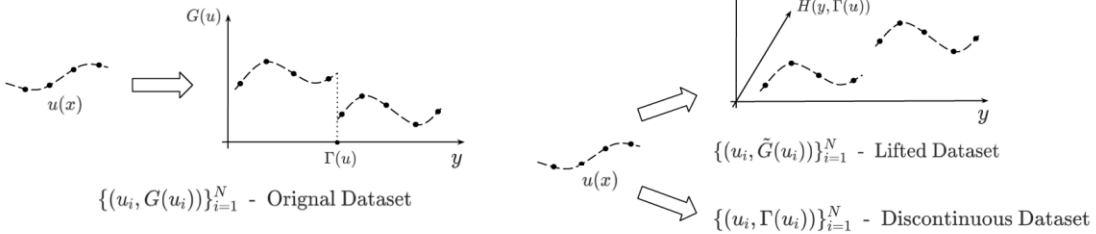
Cut-DeepONet (Our Approach)

Idea: We use Cutting Net to guide the network to ignore discontinuities and focus on learning and reconstructing the smooth regions [1].

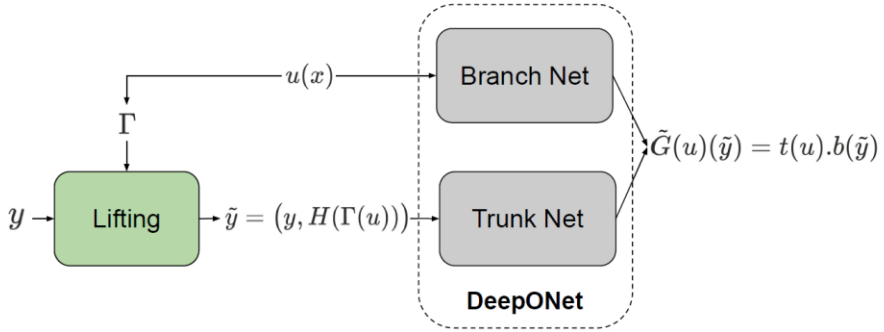
Stage 1: Cutting Net with Discontinuous Data



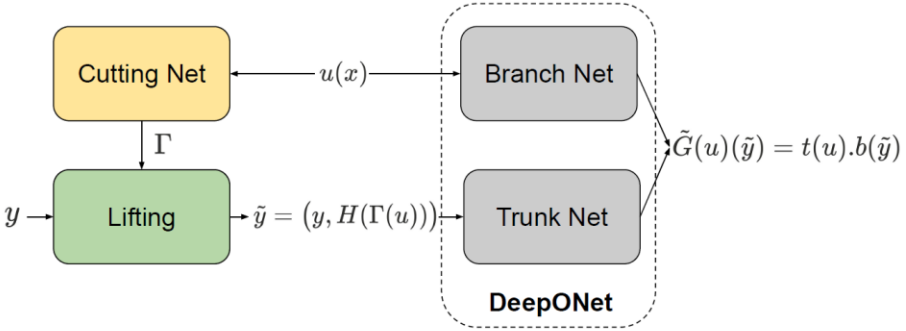
A: Training data



Stage 2: Operator Learning with Lifted Dataset

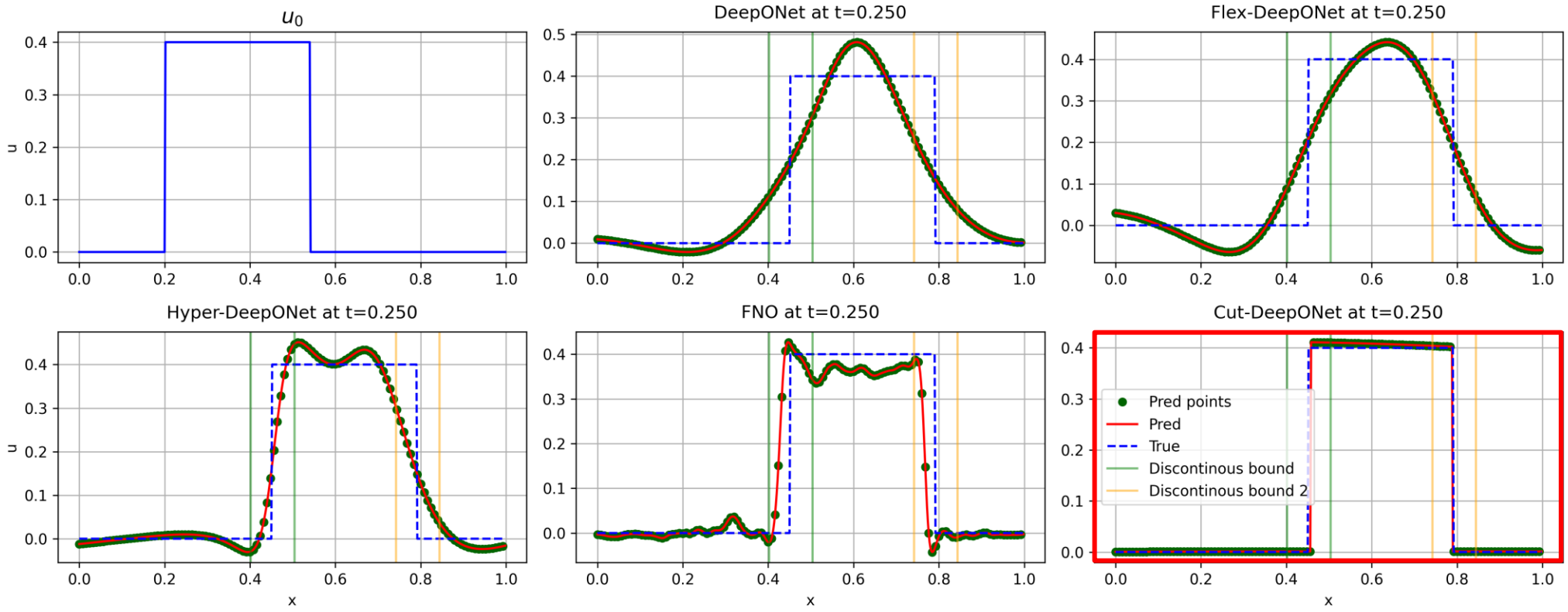


Stage 3: Cut-DeepONet Prediction



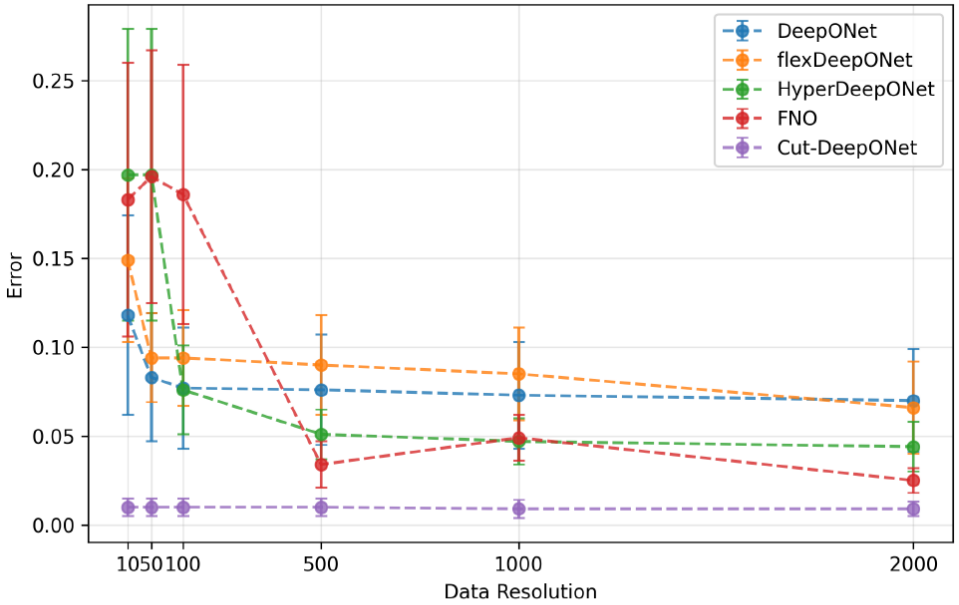
[1] Ha Dang et al. “Smooth Piecewise Cutting for Neural Operator to Handle Discontinuities and Sharp Transitions”, *2026*

Results

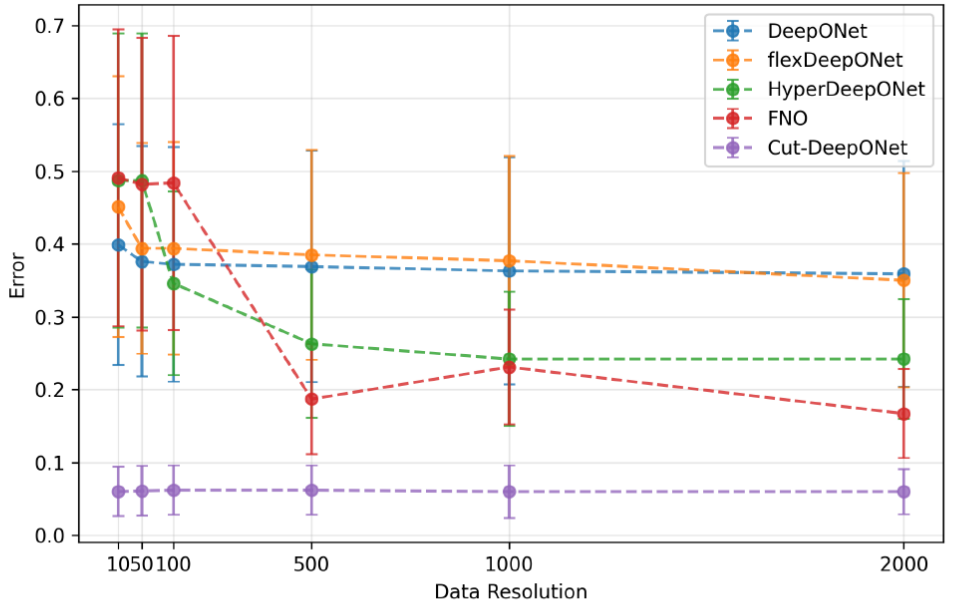


⇒ Our method can **reproduce true discontinuities** in the output function, unlike existing methods.

Results



(b) Error for different resolutions



(c) Error near discontinuities for different resolutions

⇒ By focusing only on smooth regions, the network can be **trained on low-resolution data**.

Results

		DeepONet	Flex-DeepONet	Hyper-DeepONet	FNO	Cut-DeepONet (our)
Linear Advection	L_1	0.070 ± 0.029	0.066 ± 0.026	0.044 ± 0.014	0.025 ± 0.007	$\mathbf{0.009 \pm 0.004}$
	Dis	0.359 ± 0.155	0.350 ± 0.147	0.242 ± 0.082	0.167 ± 0.061	$\mathbf{0.060 \pm 0.031}$
Inviscid Burgers'	L_1	0.089 ± 0.026	0.062 ± 0.027	0.031 ± 0.006	0.060 ± 0.035	$\mathbf{0.010 \pm 0.007}$
	Dis	0.407 ± 0.100	0.193 ± 0.035	0.135 ± 0.031	0.279 ± 0.110	$\mathbf{0.053 \pm 0.037}$
Parsimonious *(sharp jump)*	L_1	6.116 ± 1.900	9.382 ± 4.283	6.744 ± 3.095	6.287 ± 7.049	$\mathbf{2.032 \pm 0.830}$
	Dis	19.592 ± 3.335	15.077 ± 2.322	5.669 ± 2.082	$\mathbf{1.777 \pm 0.856}$	2.050 ± 0.588

Table 1: **Cut-DeepONet outperforms other methods on problems with discontinuities and sharp transitions**. *Dis* is a discontinuity-aware loss that considers only the 10% region surrounding the discontinuities.

⇒ Our method outperforms existing methods by **3x/4x across different evaluation metrics.**

Thank you



GitHub link for our paper