From Jumps to Signatures:
A Generative Method for Temporal Point Processes

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Paper

Code

1. SigTPP: Signature Matching

Temporal point processes model events in continuous time. SigTPP lifts event sequences to continuous paths and matches mean signatures of generated and observed paths. We also propose measures of fit for distributions of whole event sequences.

Interarrival Lift Φ

For events $\eta = (t_k)_{k=1}^m$ on $[0, T]$, set $t_0 = 0$ and $\tau_k = t_k - t_{k-1}$. Linearly join $(0, 0)$, (t_k, τ_k), and $(T, T - t_m)$ to form $\Phi(\eta)$.

$$X_t = \widetilde{\Phi}(\eta)_t = \big(t, \Phi(\eta)_t\big) .$$

Signature

Ordered integrals through degree M [1]:

$$S^M(X) = \big(1, S(X)^1, \dots, S(X)^M\big) ,$$

$$S(X)^k = \int_{0<u_1<\cdots<u_k<T} \bigotimes_{j=1}^{k} \mathrm{d}X_{u_j} .$$

Loss

Mean signatures [2]: $\overline{S}_\theta^M$ (model), $\overline{S}_{\text{data}}^M$ (data). Data signature standard deviation σ_{data}.

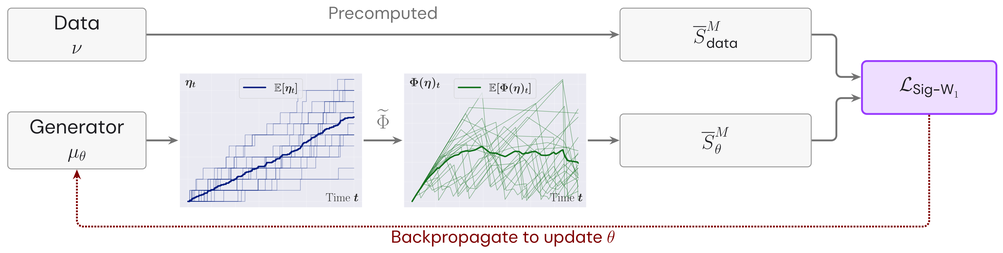
$$\mathcal{L}_{\text{Sig-W}_1} = \left\| \frac{\overline{S}_\theta^M - \overline{S}_{\text{data}}^M}{\sigma_{\text{data}}} \right\|_2^2 .$$

Relative Score

$s_{a,i,k}$: model a, dataset $i \le D_a$, metric $k \le 8$.

$$R_a = \left(\prod_{i,k} \frac{s_{a,i,k}}{s_{\text{DETER},i,k}} \right)^{1/(8D_a)} .$$

Lower is better; $R_{\text{DETER}} = 1$.



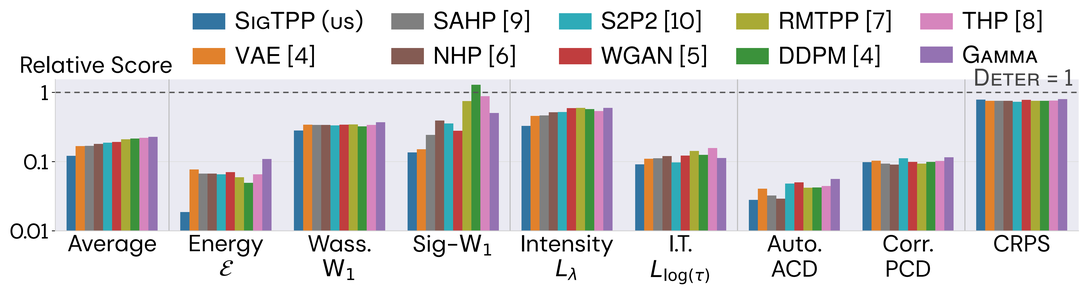
Why use Φ?

- Injective, preserving event times.
- Lipschitz in the counting-path metric.
- $1/2$-Hölder inverse for gaps $\geq \delta > 0$.

This regularity helps keep the training loss stable under small changes in event timing.

2. SigTPP: Overall Score

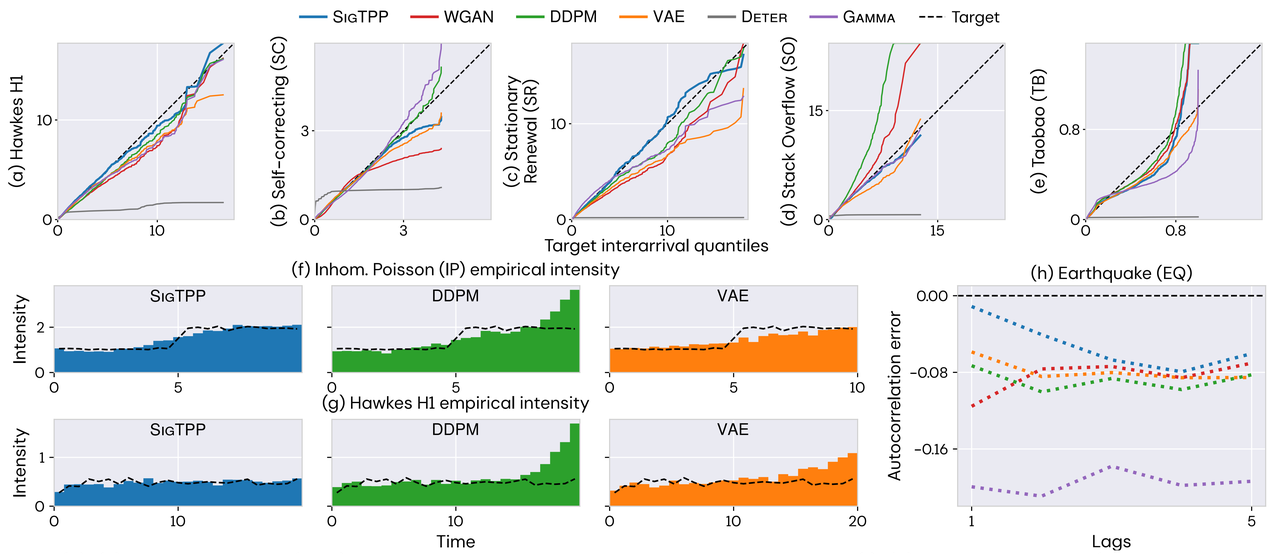
24% better (than 2nd best)	Relative score vs VAE. 12 datasets, 8 metrics, 6 models.
2.27 overall mean rank	Best overall. 12 datasets, 8 metrics, 6 models.
5/8 metrics	Lowest geometric relative score; 12 datasets, 6 models.
28% better than EasyTPP [3]	Relative score vs each baseline. 6 datasets (S2P2: 5).



11 models including Deter (dashed line), 6 datasets; S2P2: 5 (Taobao missing). Log scale.

3. Marginals, Intensity and Dependence

SigTPP captures interarrival distributions, event intensity, and autocorrelation.



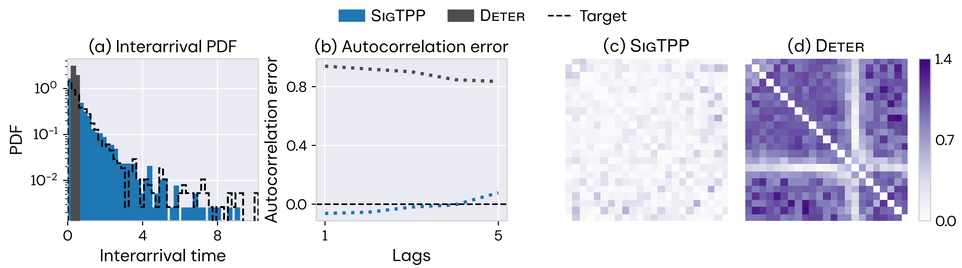
(a)–(e) Quantile–quantile (Q-Q) plots of generated against target interarrival quantiles. (f)–(g) Empirical intensity. (h) Signed autocorrelation error at lags 1–5. Zero is the target. Deter is omitted in (h) for legibility.

4. Best by MAE, Degenerate by Construction

Deter predicts a single next event, so its conditional predictive law is a Dirac measure. It has the lowest sampled mean absolute error (MAE) on all 12 datasets and the worst path W_1 on 11.

Mean ranks over 12 datasets among 6 models; lower is better (↓); 1 is best. MSE: mean squared error. Best in **bold**.

Metric	Deter	SigTPP	VAE	DDPM	WGAN	Gamma
Sampled MAE	**1.00**	4.58	2.08	4.58	4.00	4.75
MAE of median	**2.33**	4.83	**2.33**	3.00	3.58	4.92
MSE of mean	4.75	4.33	**1.58**	3.00	3.25	4.08
Energy $\mathcal{E}$	5.92	**1.50**	3.33	2.08	3.50	4.67
Path W_1	5.83	**1.50**	2.92	2.42	3.58	4.75



Yelp Airport (YLA). (a) Interarrival density on a log scale. (b) Autocorrelation error at lags 1–5, with zero as the target. (c)–(d) Absolute correlation error between interarrival positions.

MAE and MSE alone are insufficient for evaluating generative models. Evaluation also needs to measure how well generated sequences reproduce the data distribution.

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