PRIME: Poincaré Induced Measure for Learning Partial Observed Dynamical System

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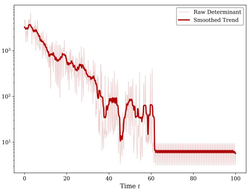
Motivation

- **Problem:** Learning dynamics from sparse partial observations is challenging:
  - Pointwise MSE may induce local contraction and lead to long-term rollout collapse.
  - Invariant-measure matching preserves global geometry but lacks long-term temporal ordering.
- **Question:** How can long-term temporal structure be incorporated into a measure-based learning framework without reverting to unreliable pointwise matching?

Key Observations



(a) Long-term rollout collapse



(b) Phase space volume contraction

Contributions

- ✓ **Long Time Rollout Collapse:** When the system is partially observed, the observed system is no longer Markovian anymore, thus pointwise MSE matching is insufficient.
- ✓ **Inefficient temporal constrain:** Measure matching framework by aligning invariant measure / delay coordinate invariant measure lacks long time temporal constraint.
- ✓ **PRIME:** Poincare return induced invariant measure incorporate recurrence-level temporal information into the measure-matching framework.

Method: PRIME

Overview

PRIME aligns the recurrence graph measure, providing a long-term temporal constraint for learning partially observed dynamical systems.

Partial Observation $y_t = h(u_t)$ → **Autoregressive Predictor** $\hat{y}_{t+1} = \Phi_\theta(y_t)$ → $\mathcal{L}_{PRIME} + \mathcal{L}_{DIM}$

- **Recurrence graph matching** $\mathcal{L}_{PRIME}$
- \+
- **Delay-invariant measure matching** $\mathcal{L}_{DIM}$

Poincaré Induced Measure

- Let Φ^t be the flow. A Poincaré section Σ is a codimension-one hypersurface transverse to the flow. The Poincaré return map is

$$P(u) := \Phi^{\tau(u)}(u), \qquad \tau(u) := \inf\{t > 0 : \Phi^t(u) \in \Sigma\}$$

- The induced measure ν_Σ characterizes the long-run distribution of return states on Σ for ν_Σ-almost every initial return state.

Recurrence Graph Measure

- Given return states $\{y_i\}$ on Σ, construct the recurrence coordinate $\boldsymbol{r}_i = [\boldsymbol{y}_i, \boldsymbol{z}_i]$ where z_i is the time-delay component. The recurrence graph measure is defined as

$$\pi_\star := \mathcal{L}(\boldsymbol{r}_i, \boldsymbol{r}_{i+1})$$

- For a recurrence block B_k of L consecutive recurrence event are $\{r_k, r_{k+1}, \dots, r_{k+L-1}\}$, the observed data and model-generated recurrence graph measures are:

$$\hat{\pi}_d^k = \frac{1}{L-1}\sum_{j=k}^{k+L-2} \delta(\boldsymbol{r}_j^{\boldsymbol{d}}, \boldsymbol{r}_{j+1}^{\boldsymbol{d}}) \quad \hat{\pi}_\theta^k = \frac{1}{L-1}\sum_{j=k}^{k+L-2} \delta(\boldsymbol{r}_j^{\boldsymbol{\theta}}, \boldsymbol{r}_{j+1}^{\boldsymbol{\theta}})$$

Training Objective

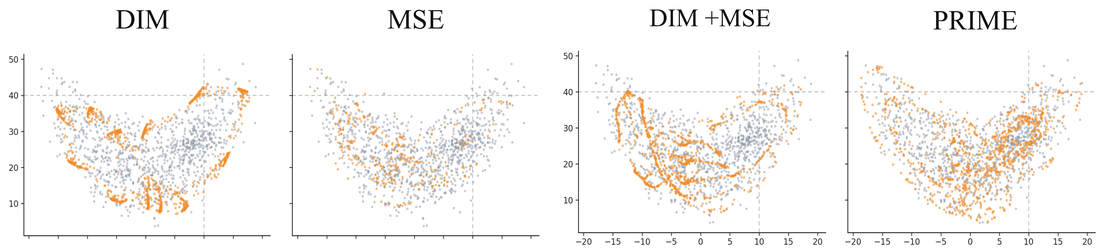
- The PRIME loss is defined as:

$$\mathcal{L}_{PRIME} = \mathbb{E}_{B_k}\left[\frac{1}{L}\left[\mathcal{D}(\hat{\pi}_\theta^k, \hat{\pi}_d^k) + \omega_0 \mathcal{D}(\hat{\mu}_{\theta,0}^k, \hat{\mu}_{d,0}^k) + \omega_1 \mathcal{D}(\hat{\mu}_{\theta,0}^k, \hat{\mu}_{d,1}^k)\right]\right]$$

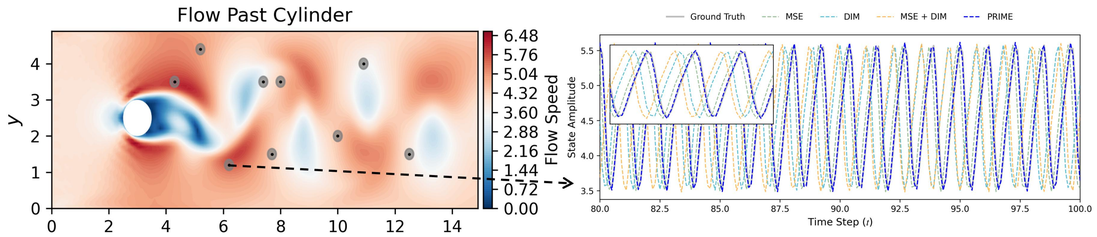
where $\hat{\mu}_{v,c}^k = \frac{1}{L-1}\sum_{j=k}^{k+L-2} \delta_{j+c}^v$ for $v \in \{d, \theta\}, c \in \{0,1\}$denotes the corresponding source and target marginals.

Results

Qualitative Results



Unrolled trajectory for the X-Z partially observed Lorenz63.



Forecasting the dynamics of noisy observations in a Kármán vortex street.

Quantitative Results

System	Model	Short- & long-term NMSE (↓)			Structure Metrics (↓)	
		NMSE 20	NMSE 100	NMSE 400	SD	Lag-PSD
Lorenz63	**PRIME**	**0.20 ± 0.13**	0.29 ± 0.062	0.32 ± 0.039	**8.11 ± 10.43**	**0.74 ± 0.29**
	MSE	0.20 ± 0.13	**0.29 ± 0.060**	**0.28 ± 0.040**	53.06 ± 31.61	3.85 ± 3.38
	DIM	0.26 ± 0.16	0.35 ± 0.083	0.37 ± 0.066	21.57 ± 21.10	1.81 ± 2.092
	DIM+MSE	0.21 ± 0.12	0.28 ± 0.068	0.29 ± 0.054	31.91 ± 26.83	1.91 ± 2.12
KS	**PRIME**	**1.30 ± 0.52**	**1.46 ± 0.23**	**1.47 ± 0.12**	**1.99 ± 0.78**	1.66 ± 1.24
	MSE	1.92 ± 0.87	7.21 ± 30.14	$(2.57 \pm 9.88) \times 10^2$	$(1.19 \pm 3.92) \times 10^4$	1.59 ± 0.25
	DIM	1.72 ± 0.61	2.22 ± 0.93	16.03 ± 46.36	$(1.20 \pm 2.73) \times 10^3$	**0.96 ± 0.63**
	DIM+MSE	1.68 ± 0.78	2.52 ± 1.77	11.70 ± 31.21	$(3.36 \pm 9.46) \times 10^2$	1.48 ± 0.85
CFD	**PRIME**	**(6.39±2.91)× 10^{-5}**	**(6.91±3.04)**× 10^{-5}	**(1.69±1.17)**× 10^{-4}	**(3.91±0.39)**× 10^{-3}	**0.74 ± 0.12**
	MSE	(1.12±0.62)×10^{-4}	(3.80±3.85)×10^{-4}	(4.37±5.02) × 10^{-3}	(5.81±0.72) × 10^{-3}	1.33 ± 0.60
	DIM	(1.32±1.16)×10^{-4}	(2.06±2.73)×10^{-3}	0.02±0.02	(1.63±0.61)×10^{-3}	1.38 ± 0.43
	DIM+MSE	(8.60±4.14)×10^{-5}	(9.92±5.36)×10^{-4}	0.01±0.01	(4.02±0.02)×10^{-3}	1.32 ± 0.34

Conclusion

- PRIME bridges the gap between pointwise matching and invariant-measure matching by imposing long-term temporal constraints through recurrence-level temporal organization.
- It effectively mitigates long-term rollout collapse and improves temporal consistency while maintaining or enhancing long-term geometric and spectral fidelity.

Code: github.com/CrispDyt/2026-NeurIPS-PRIME