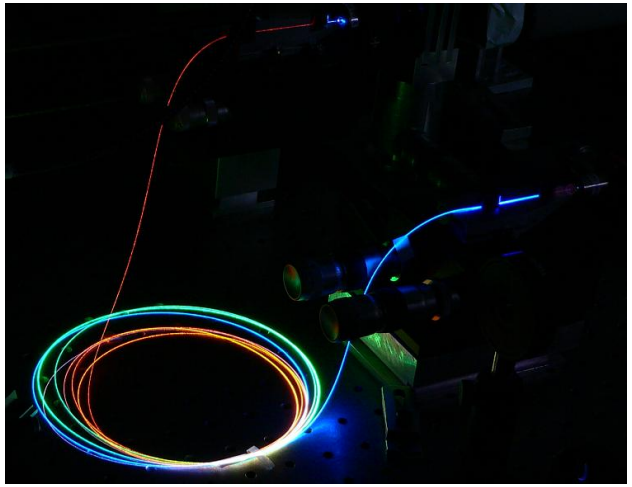
NEURAL INFORMATION PROCESSING SYSTEMS

Linear Generator Networks for Nonlinear Partial Differential Equations

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USC Viterbi

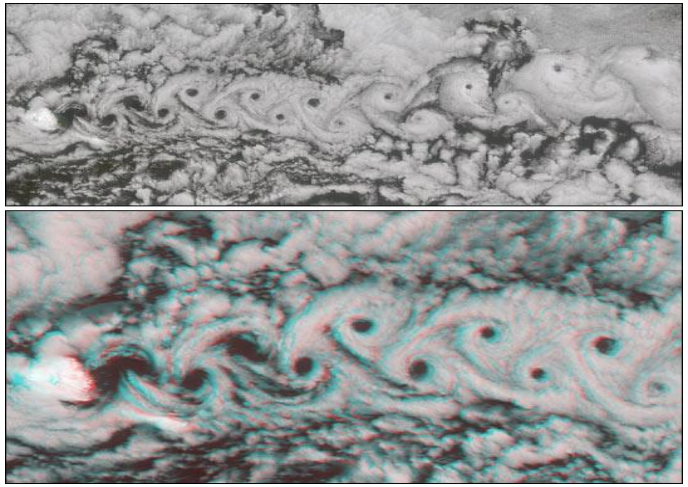
Nonlinear dynamics shape our world



Light propagation in Optical fiber



Chemical reaction propagating across a petri dish



A Vortex Street in the clouds of Arctic

AI can predict how these systems evolve. Engineering needs the underlying rules.

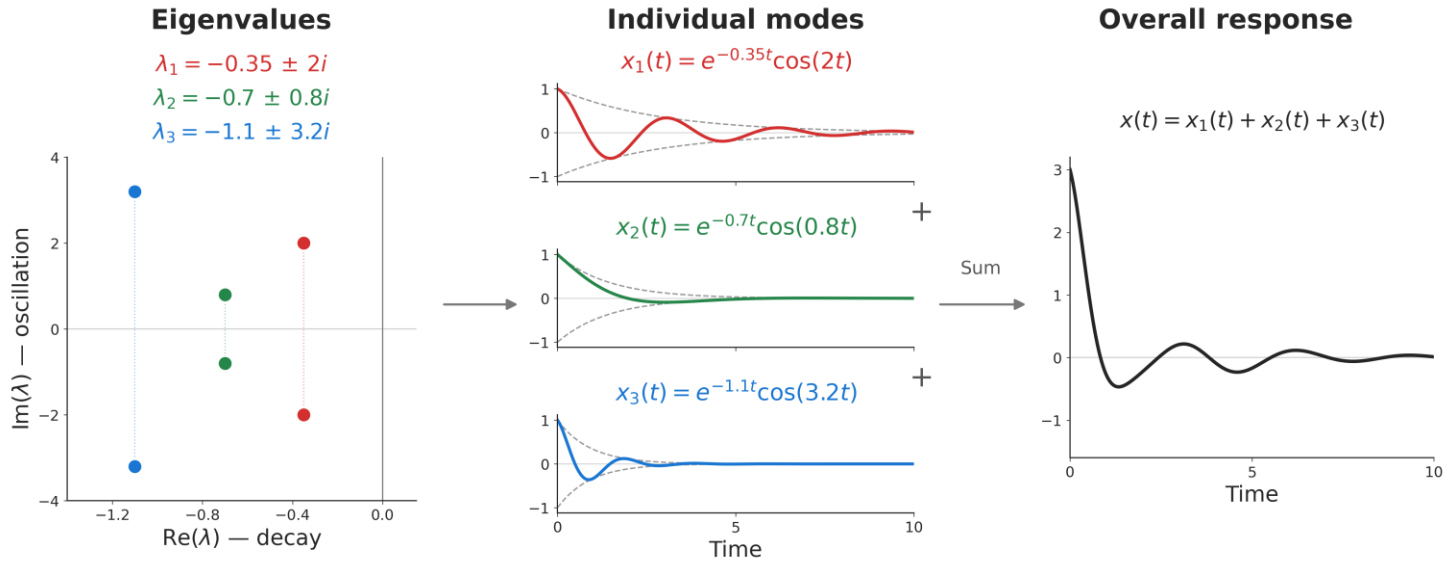
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NASA/GSFC/LaRC/JPL, MISR Team

Linear Dynamics are Interpretable

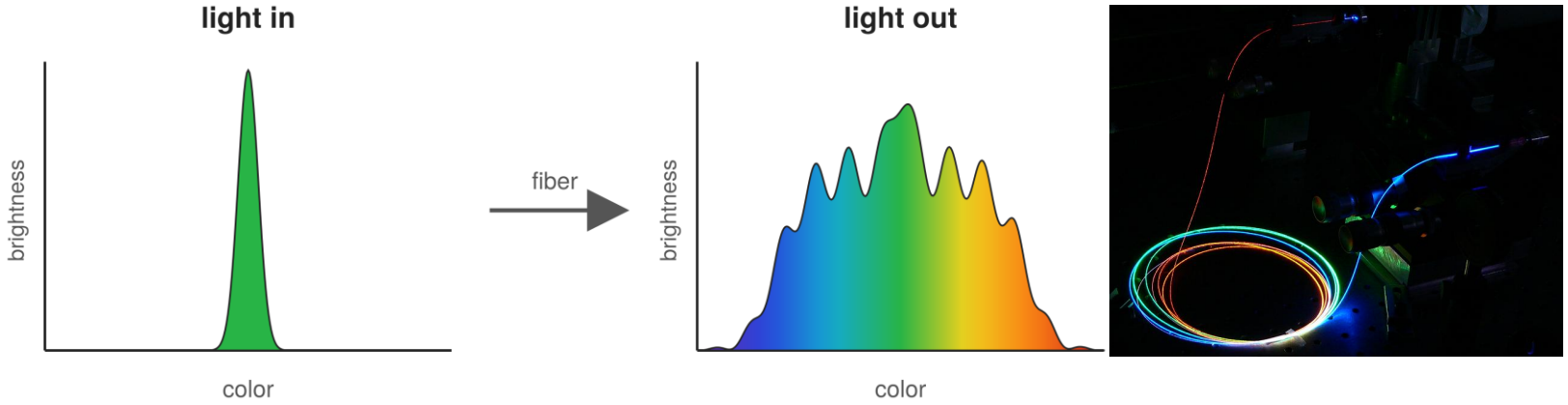
Linear dynamics can be understood through independently evolving modes. Nonlinear ones don't.

Linear: each mode oscillates and decays on its own. The whole is just the sum of the modes.

$$\frac{dx}{dt} = \underbrace{\mathrm{L}}_{\text{generator}} x$$



Nonlinear: The modes interact nonlinearly and create new modes

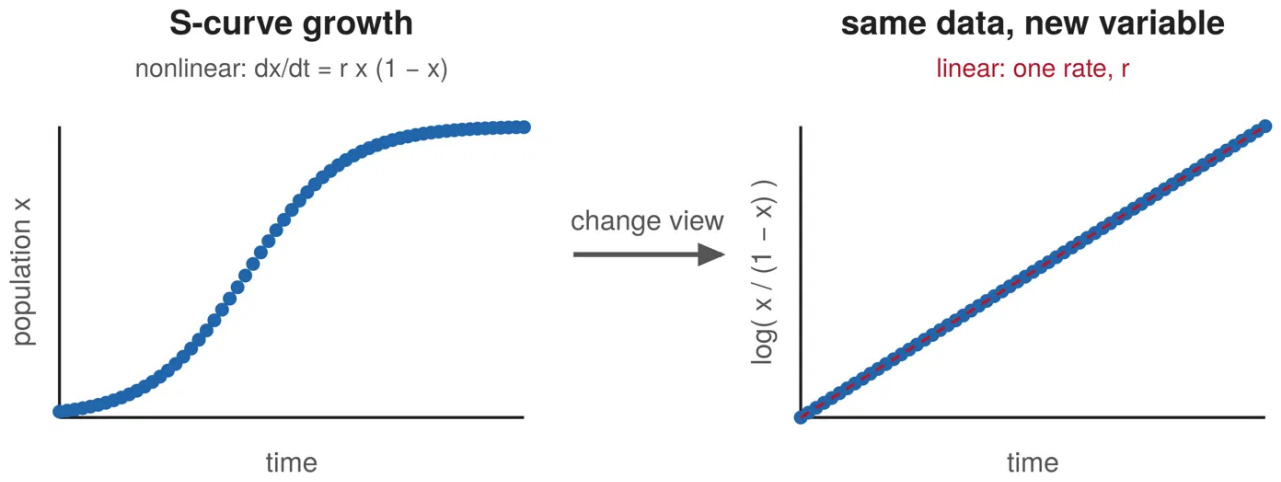


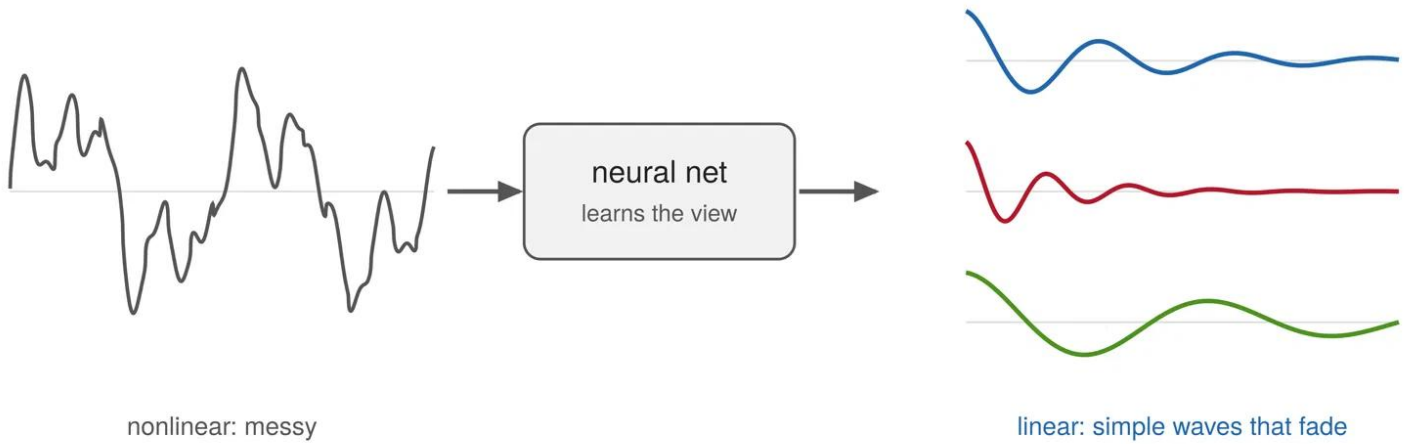
Turn Nonlinear to Linear (approx.) Dynamics

Koopman Theory: In a lifted, higher-dimensional space, nonlinear dynamics become (approximately) linear.

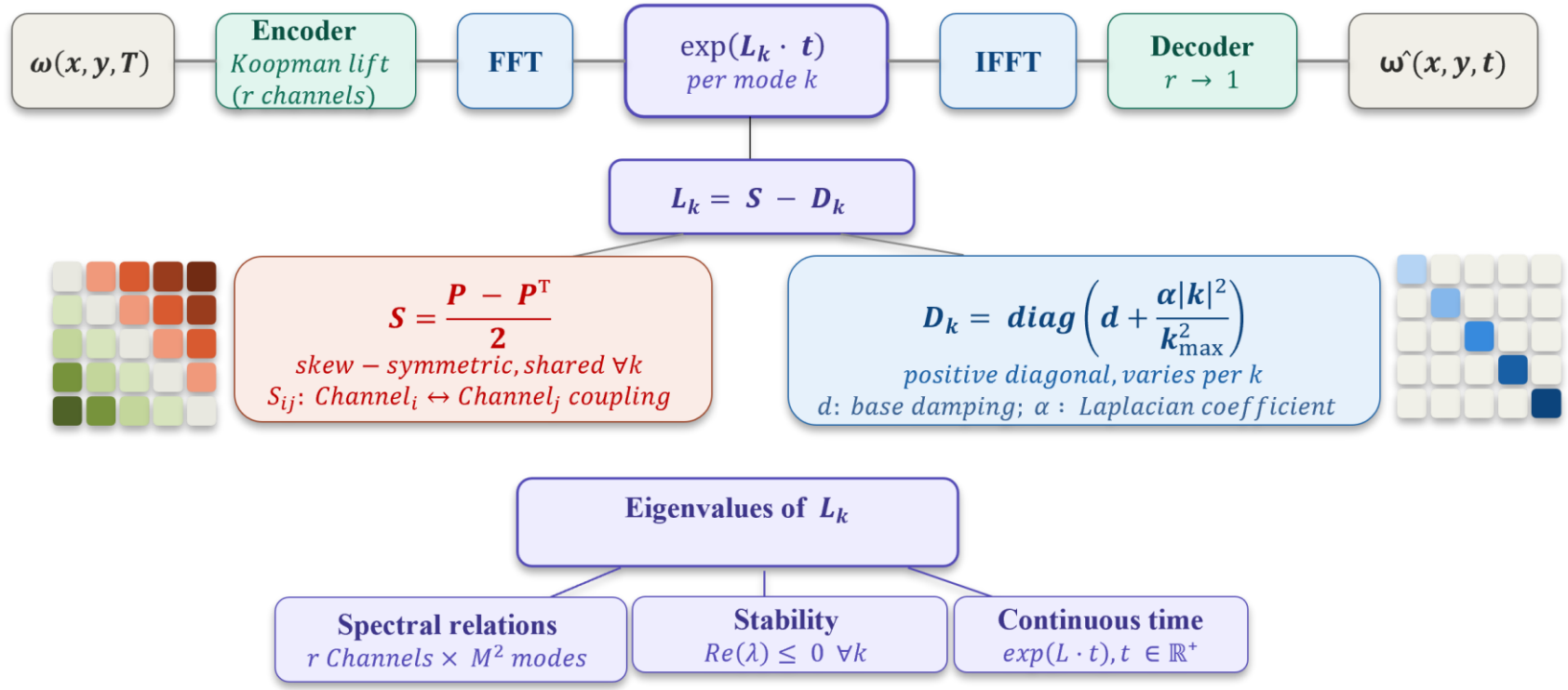
Our idea:

- ❑ Learn the Nonlinear to linear lift
- ❑ Inside the lifted coordinate learn the generator.



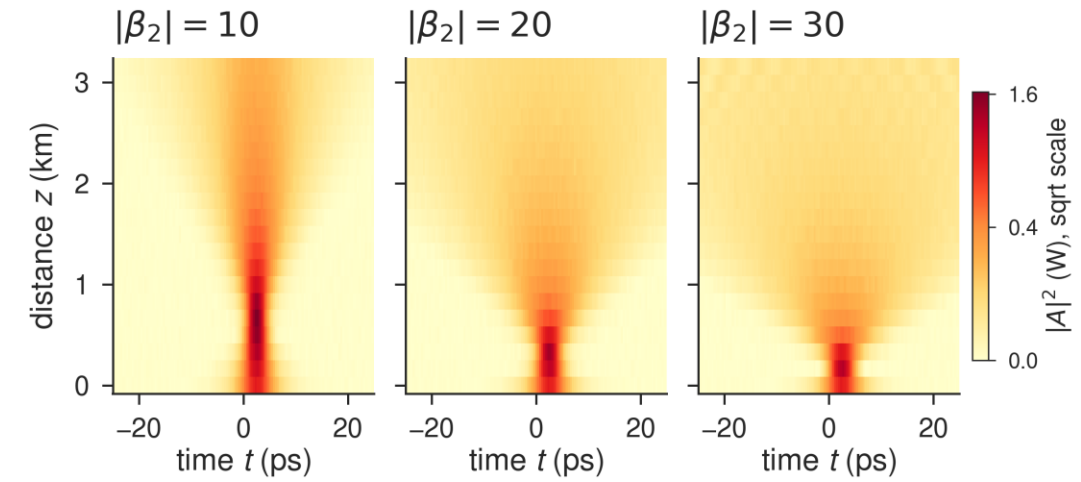


Linear Generator Networks (LGN): Architecture



LGN Recovers Optical dispersion

The learned rotation frequencies of S matrix estimate dispersion within 3.1%, with no fitted constant.



read β_2 from the fastest rotation

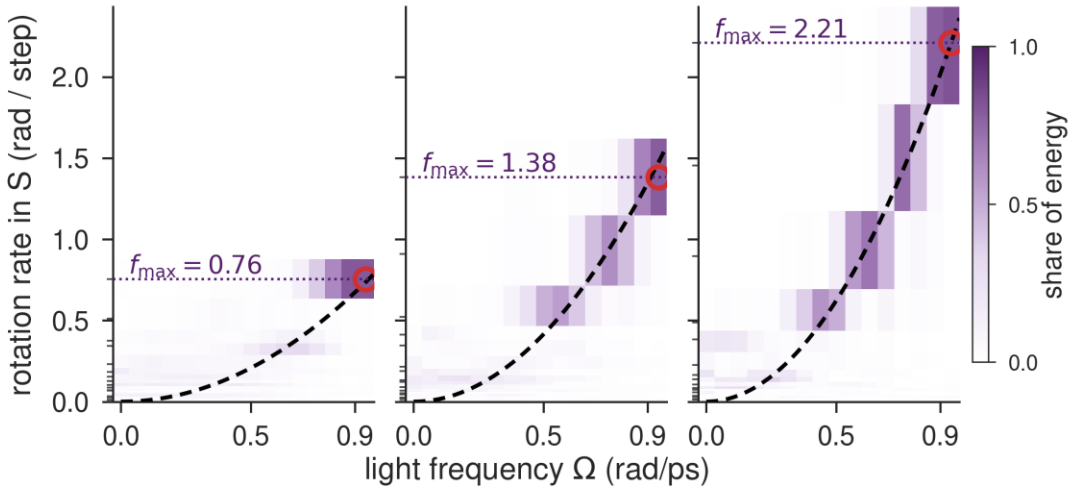
$$f_{\max} = \frac{|\beta_2|}{2}\,\Omega_{15}^2\,\Delta z$$

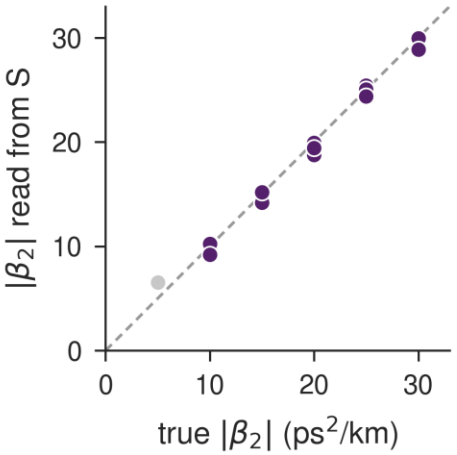
$$|\beta_2| = \frac{2 f_{\max}}{\Omega_{15}^2\,\Delta z} = 13.56\, f_{\max}$$

read: 10.2 18.8 30.0

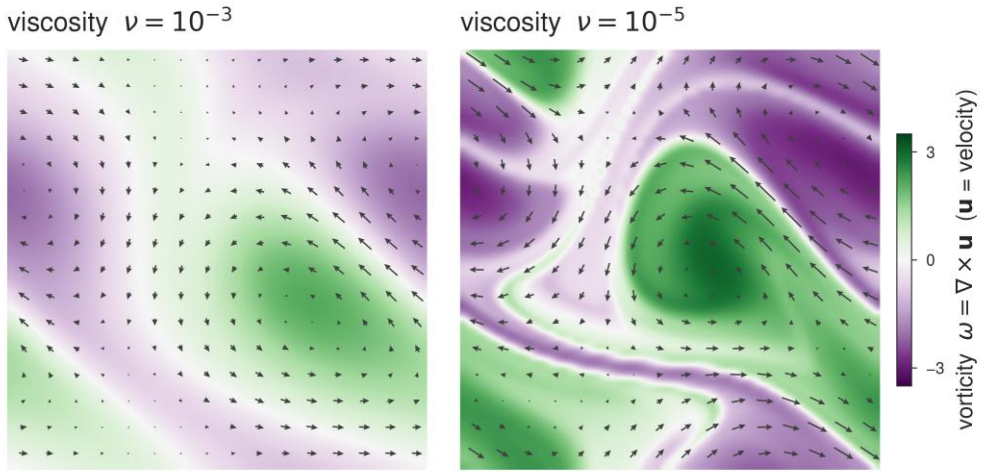
true: 10 20 30

$\Omega_{15} = 0.94$ rad/ps and $\Delta z = 0.166$ km from the grid



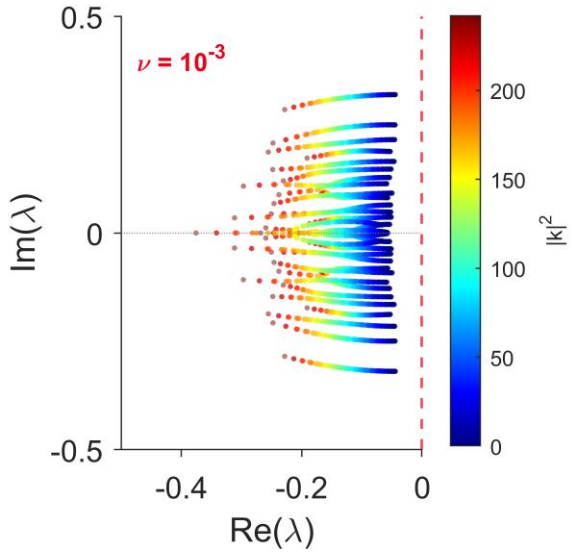


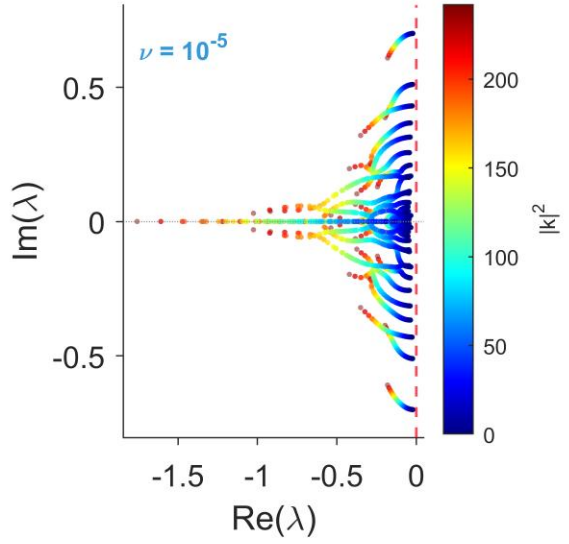
Reading the spectrum of Turbulent Fluid



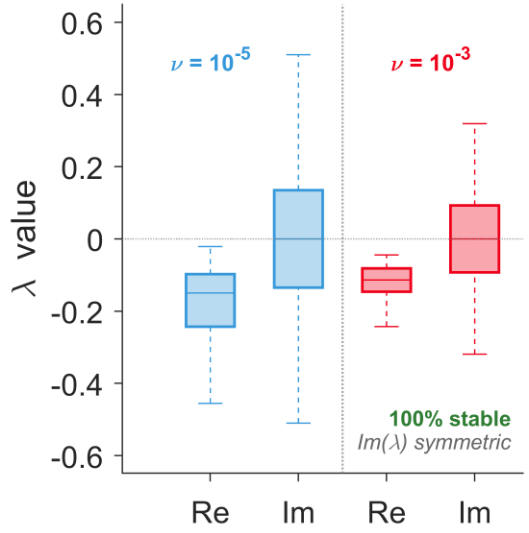
← Faster decay

← Higher Oscillation frequency





Lower viscosity produces finer flow structures and a broader learned frequency spectrum.



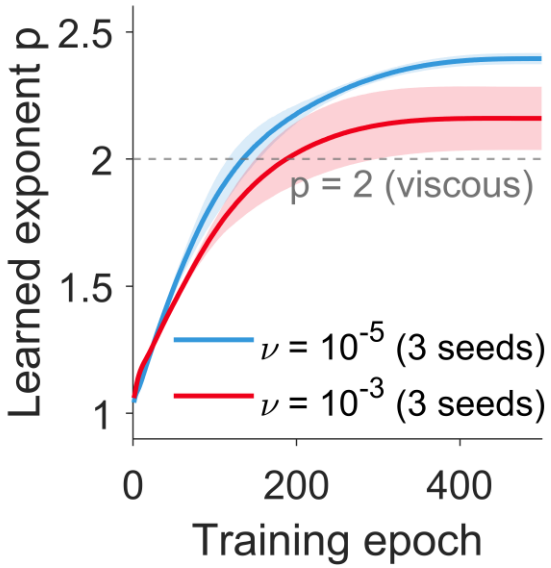
Physics From the Generator

Navier–Stokes in vorticity form

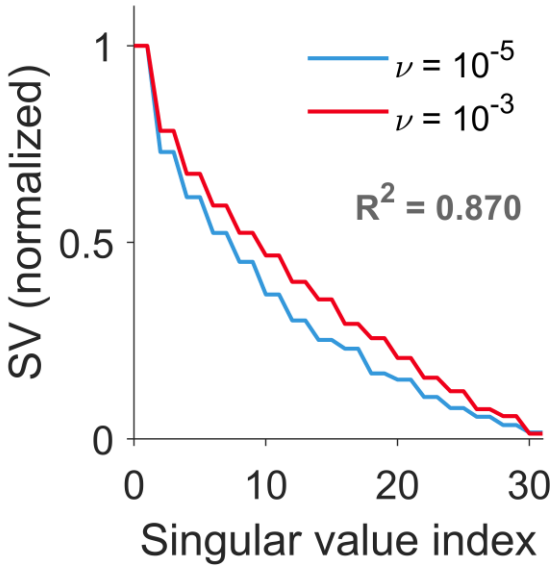
$$\underbrace{\partial_t \omega}_{\text{evolution}} + \underbrace{\mathbf{u}\cdot\nabla\omega}_{\text{nonlinear transport}} = \underbrace{\nu\nabla^2\omega}_{\text{viscous damping}} + \underbrace{f}_{\text{forcing}}, \qquad \nabla\cdot\mathbf{u} = 0.$$

After a spatial Fourier transform

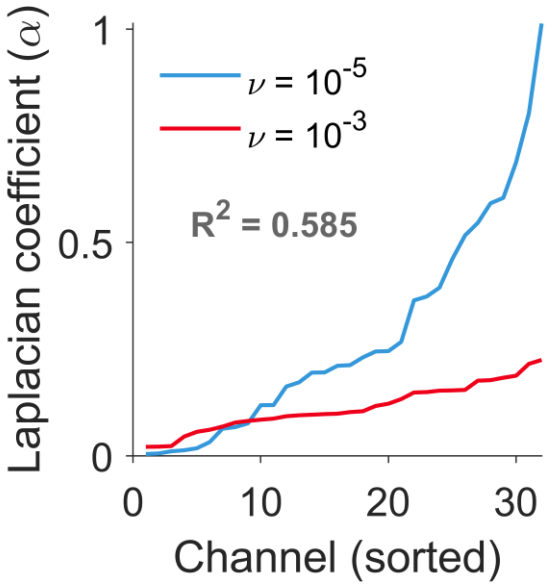
$$\frac{d\hat{\omega}_{\mathbf{k}}}{dt} = \underbrace{-\widehat{\mathbf{u}\cdot\nabla\omega}_{\mathbf{k}}}_{\text{coupling between modes}} \underbrace{-\nu|\mathbf{k}|^2\hat{\omega}_{\mathbf{k}}}_{\text{scale-dependent damping}} + \hat{f}_{\mathbf{k}}.$$



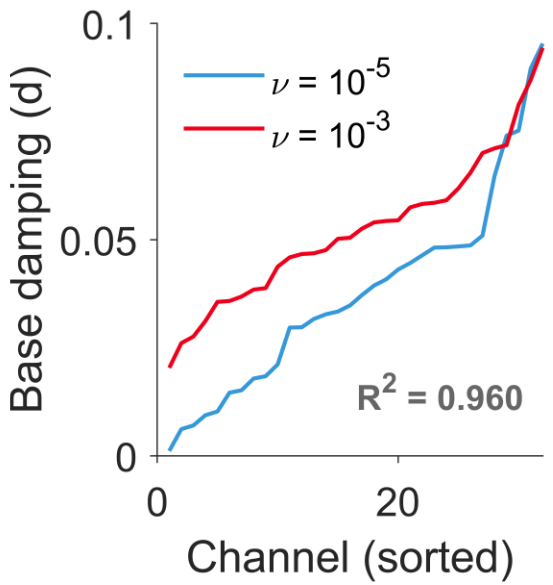
Learned damping exponent near 2



Similar normalized coupling spectra

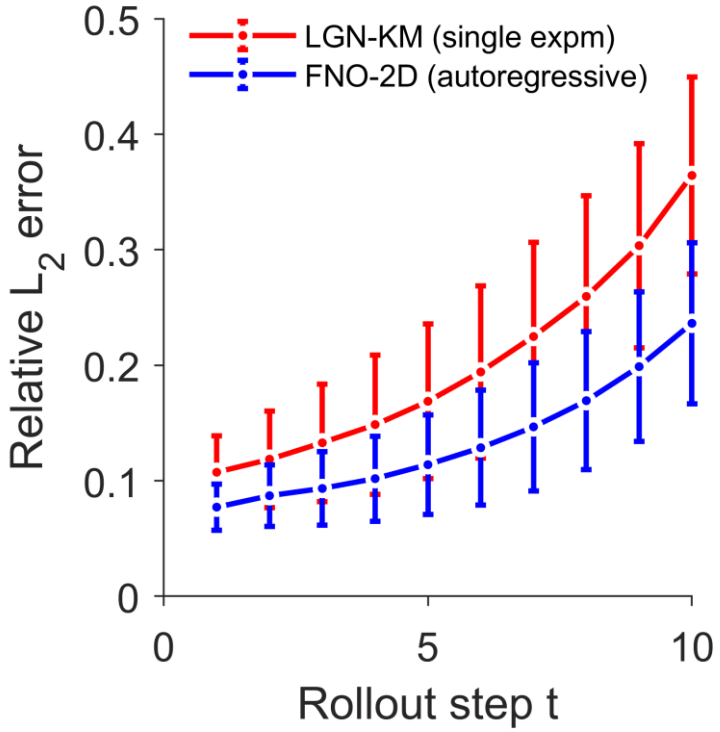


Mode dependent Laplacian coefficient

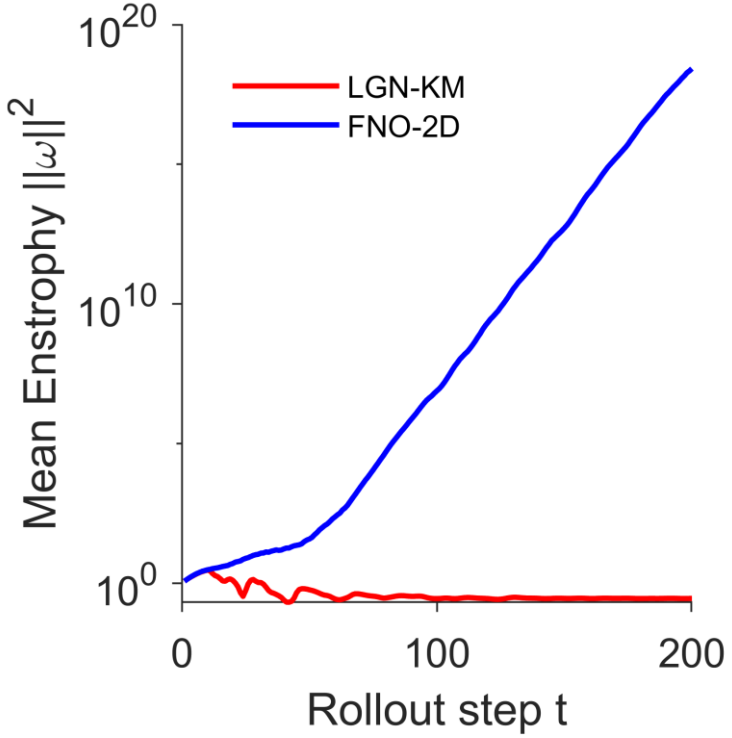


Wavenumber-independent base damping

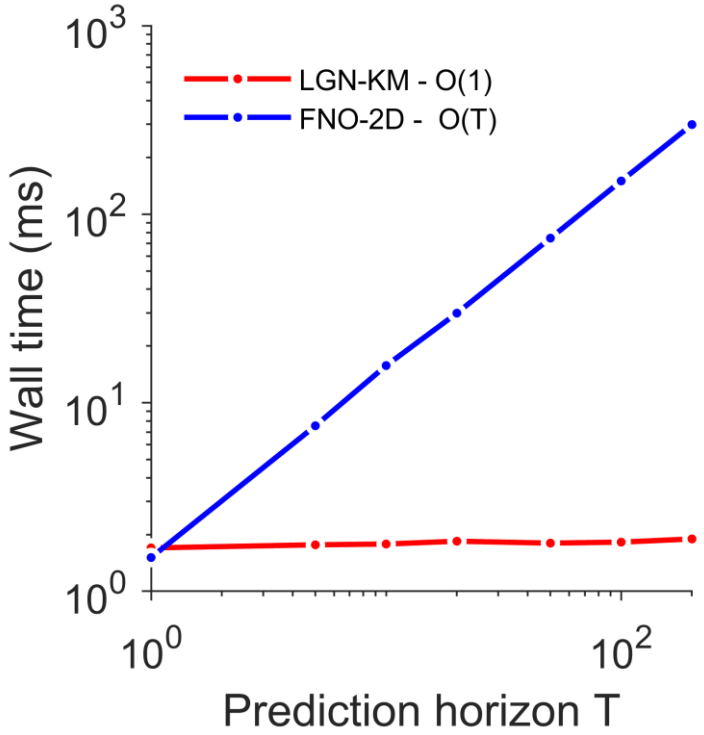
Stable, Direct Propagation



Lower short horizon accuracy

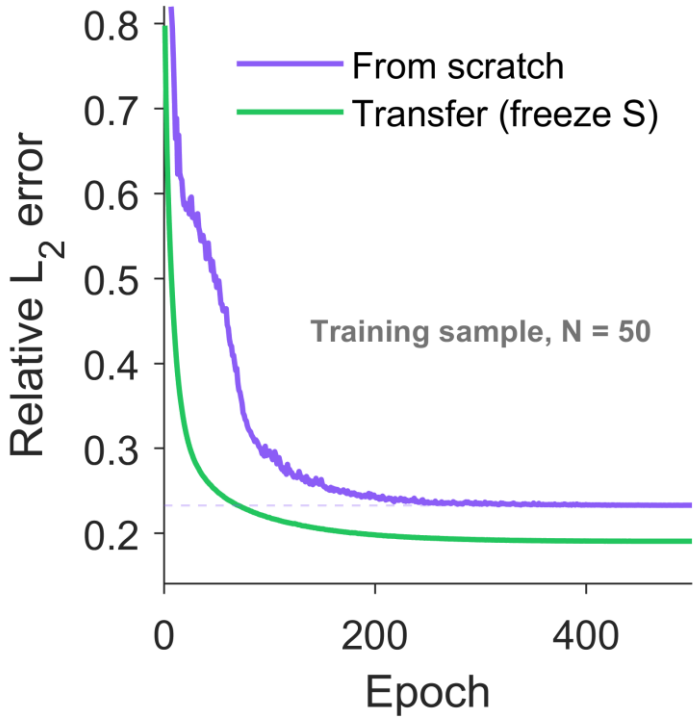


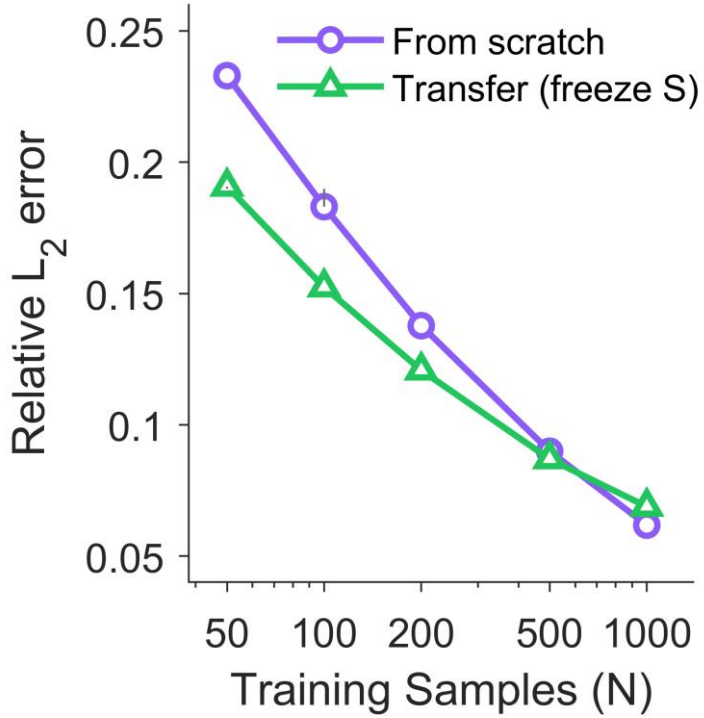
Stable by construction

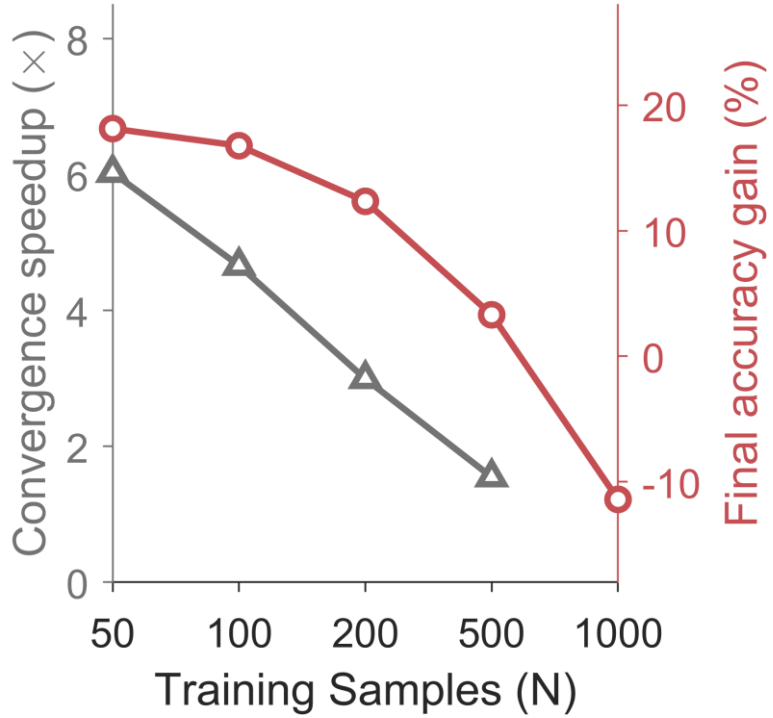


Matrix-exponential evaluation gives nearly constant prediction time

Physics Informed Model Transfer







- ❑ S is frozen; the encoder, decoder and D are fine-tuned.
- ❑ Transfer accelerates convergence and reduces prediction error at small training budgets
- ❑ its advantage decreases as more target data become available

Learning a structured generator connects prediction with interpretable dynamical information.

https://github.com/ShafayethJamil/Linear-Generator-Network-Koopman

Limitations

- ❑ One linear generator per model; can't represent multiple stable states.
- ❑ Contractive by design; can't sustain forcing, so long rollouts decay to the mean.
- ❑ Dense matrix exponentiation scales as $O(r^3)$ per Fourier mode.

Future directions

- ❑ Forcing input: ż = Lz + Bu
- ❑ Multi generator system for multi attractor systems.
- ❑ Time dependent generator for modeling periodically modulated systems.
- ❑ Coupled generators for modeling multiphysics systems.

Acknowledgements



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AIR FORCE RESEARCH LABORATORY



Thanks