

3D Fresnel Volumizing for Efficient Implicit Velocity Field Reconstruction

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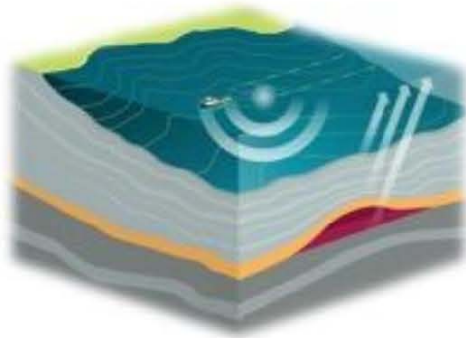
Background

- **Internal velocity reconstruction** is a prevalent scientific problem

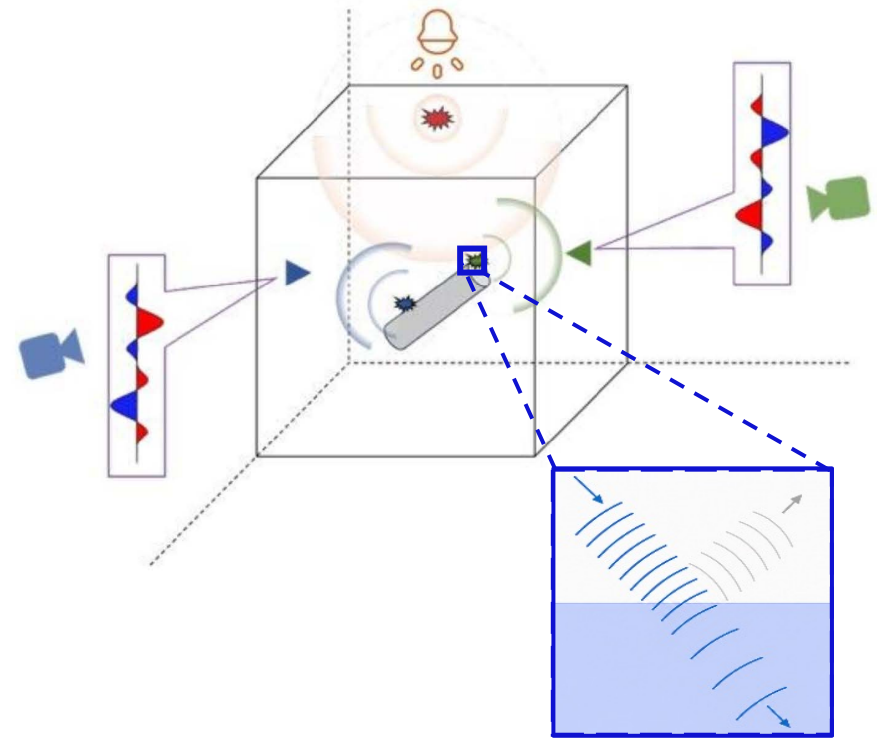
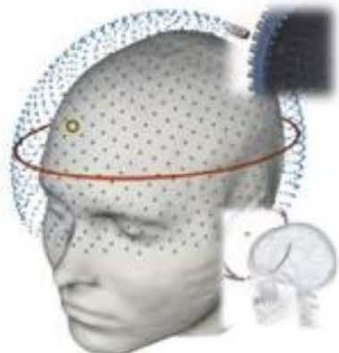
Non-destructive testing



Geological exploration



Medical imaging

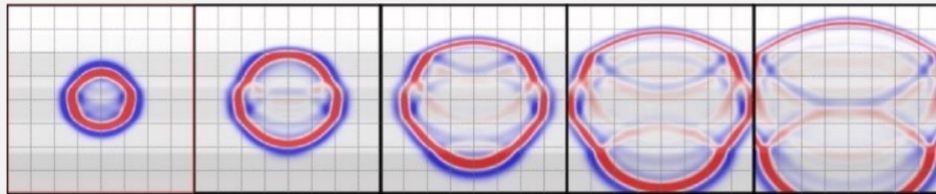


Reflection & Transmission

Background

- Wave theory: Full waveform inversion

◆ PDE: Wave equation

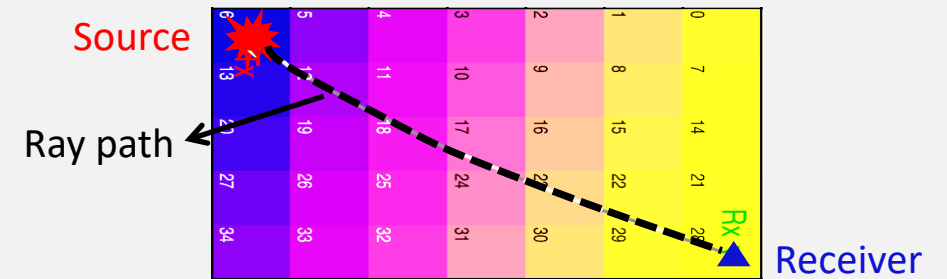


Time

- ◆ Leveraging full wavefield information
- ◆ High accuracy but low efficiency

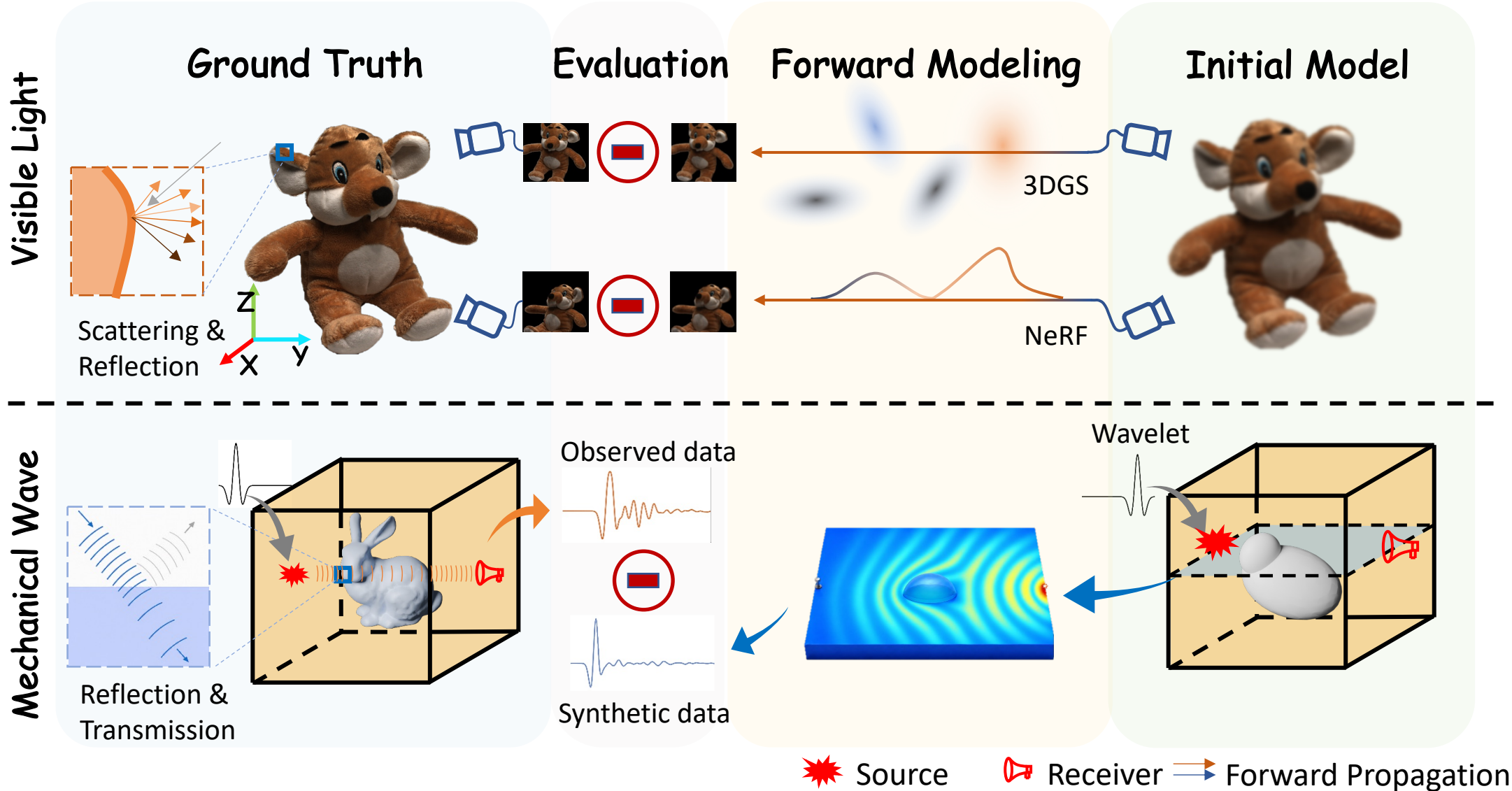
- Ray theory: Traveltime Tomography

◆ PDE: Eikonal equation



- ◆ Leveraging first-arrival traveltime
- ◆ High efficiency but low accuracy

Background



Background

Internal velocity reconstruction

vs.

Appearance property reconstruction

- More severe ill-posedness
- More complex forward modeling
- More sparse observations



- **Implicit neural representation** remain highly valuable for internal reconstruction tasks

Motivation

- Our goal: trade-off between **performance** and **efficiency** in **3D** tasks

Previous studies:

- Physics-informed robust and implicit full waveform inversion without prior and low-frequency information.
- Implicit seismic full waveform inversion with deep neural representation.
- Unveiling the mechanism of continuous representation full-waveform inversion: A wave based neural tangent kernel framework.

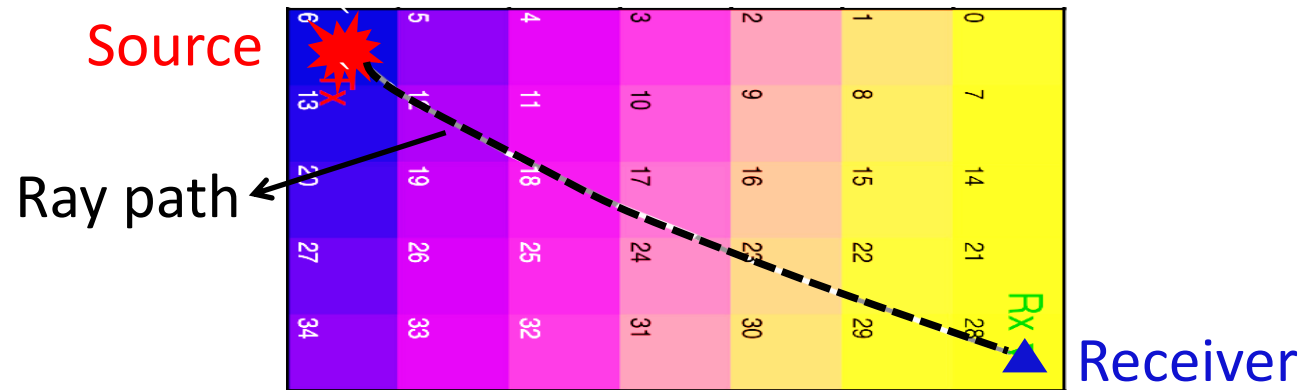
- Implicit neural representation + Full waveform inversion
(Wave equation)

✗ High computational time complexity

✗ High computational memory complexity

Motivation

- Alternative: Implicit neural representation + Traveltime Tomography
(Eikonal equation)

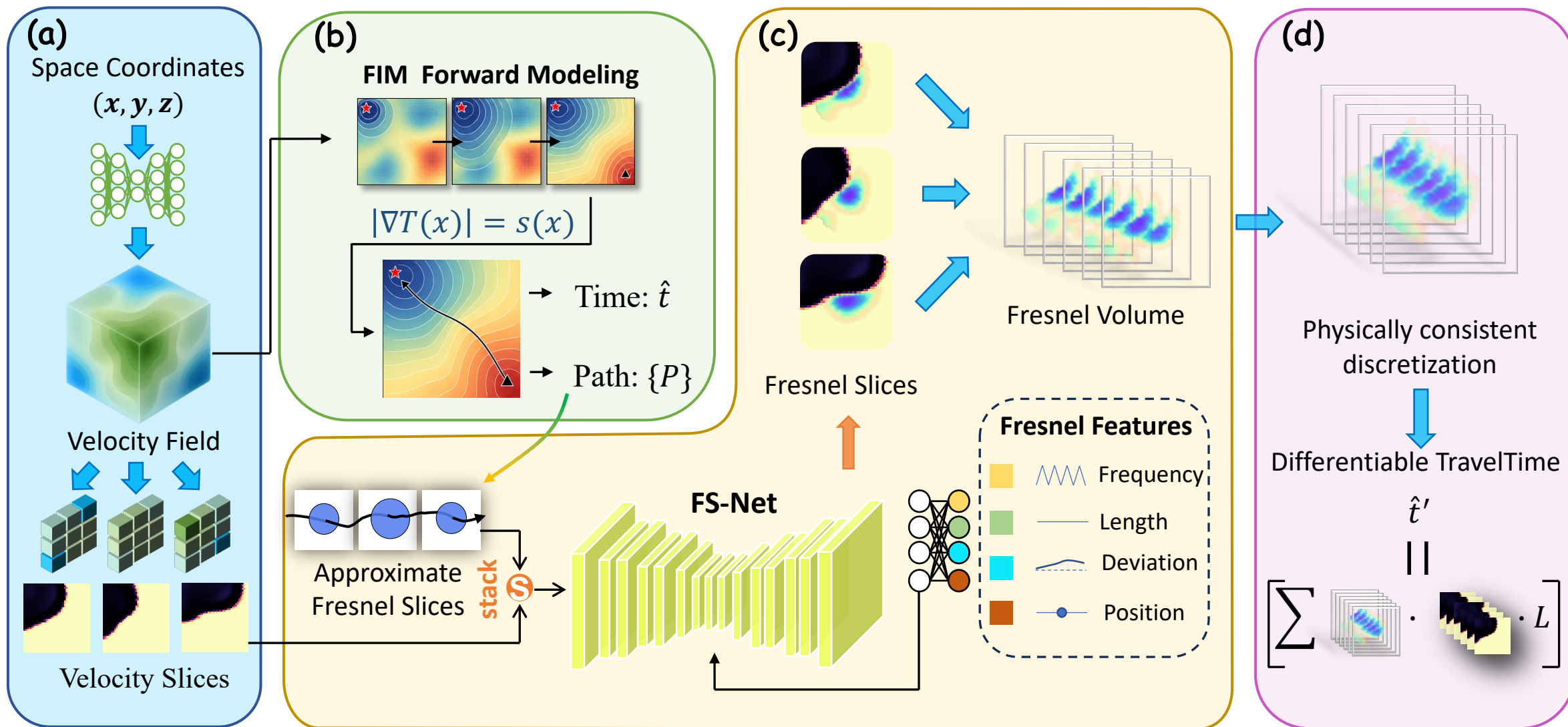


Forward traveltimes: $T_i = \sum_j s_j L_{ij}$

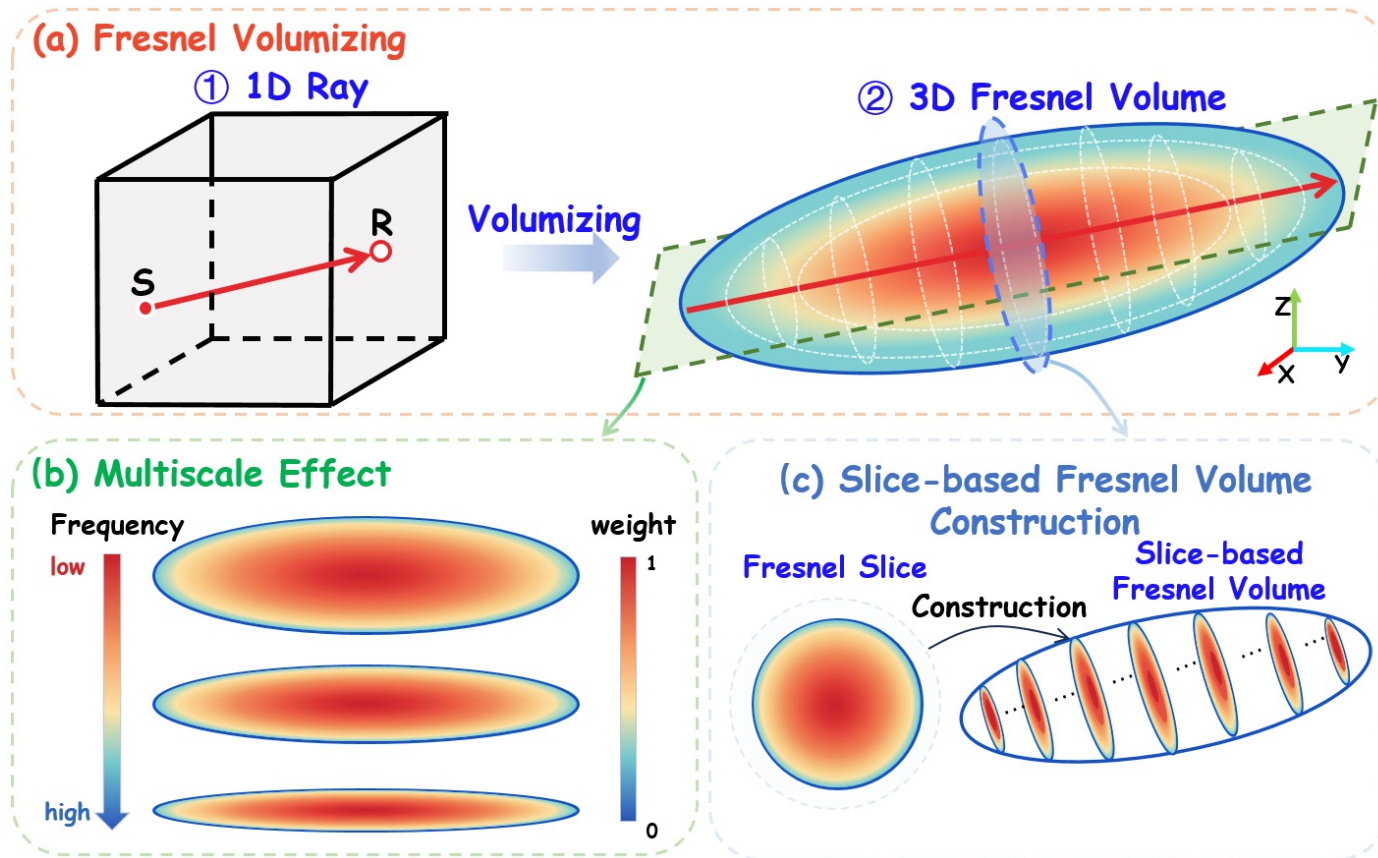
Backward gradient: $\frac{\partial \mathcal{L}}{\partial s_j} = \frac{\partial \mathcal{L}}{\partial T_i} \frac{\partial T_i}{\partial s_j} = \frac{\partial \mathcal{L}}{\partial T_i} L_{ij}$

- Sparse gradient
- Lack of multiscale capability
- Large-scale forward modeling

Pipeline



Gradient diffusion via Fresnel volumes



- Fresnel volume definition:

$$|T_{S\mathbf{x}} + T_{\mathbf{x}R} - T_{SR}| \leq 1/2f$$

$$\Delta T(\mathbf{x}) = T_S(\mathbf{x}) + T_R(\mathbf{x}) - T_{SR}$$

$$w = \cos^2\left(\frac{\pi}{2}\Delta T f\right)$$

Gradient diffusion via Fresnel volumes

- Forward traveltimes

$$T_i = \sum_j \sum_k \bar{w}_k^{ij} s_k^{ij} L_{ij}, \quad \text{where } \bar{w}_k^{ij} = \frac{w_k^{ij}}{\sum_k w_k^{ij}}$$

- Backward gradient

$$\frac{\partial \mathcal{L}}{\partial s_k^{ij}} = \frac{\partial \mathcal{L}}{\partial T_i} \frac{\partial T_i}{\partial s_k^{ij}} = \frac{\partial \mathcal{L}}{\partial T_i} \bar{w}_k^{ij} L_{ij}, \quad \text{where } \bar{w}_k^{ij} = \frac{w_k^{ij}}{\sum_k w_k^{ij}}$$

Slice-based Fresnel volume construction

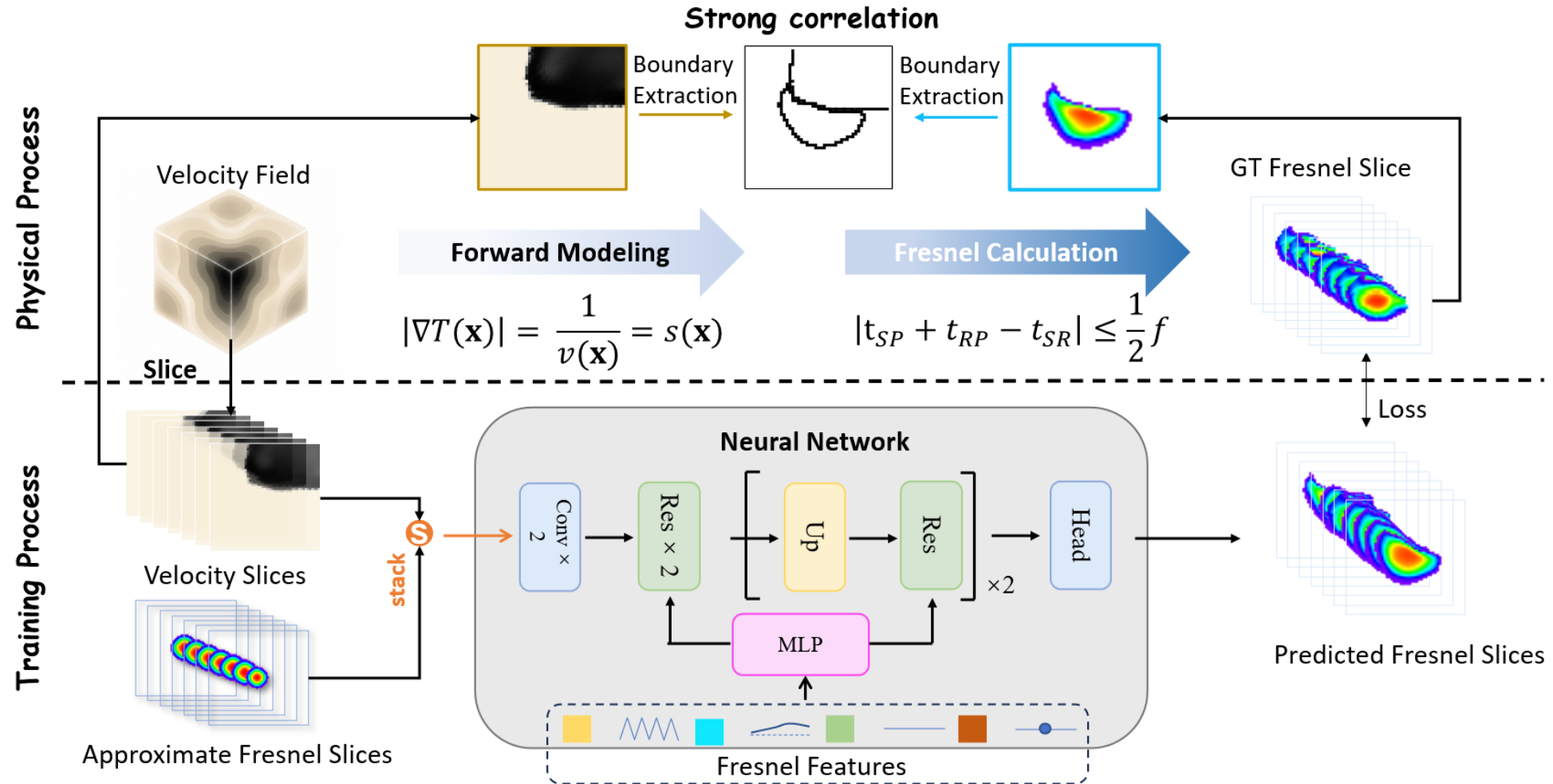
- Physical definition

$$|T_{S\mathbf{x}} + T_{\mathbf{x}R} - T_{SR}| \leq 1/2f$$

- Paraxial approximation

$$r(\mathbf{x}) = \sqrt{\frac{d(L-d)}{fLs(\mathbf{x})}} \quad (0 \leq d \leq L)$$

Slice-based Fresnel volume construction



- only **8,317** parameters

Experimental settings

- **Dataset:** ABC, ShapeNetSem, and Thingi10k
- **Resolution:** $128 \times 128 \times 128$
- **Background velocity:** 3 km/s **Target velocity:** 1 km/s
- **Observation:** 3 groups, 9×9 through-transmission setups
- **Baselines:** FWI-based Methods
- **Hardware:** 2 NVIDIA A100 GPUs

Quantitative results

- Standard condition

	Duck			Gerotor			Seal			Geometry			Bunny			
	PSNR ↑	SSIM ↑	Time ↓	PSNR ↑	SSIM ↑	Time ↓	PSNR ↑	SSIM ↑	Time ↓	PSNR ↑	SSIM ↑	Time ↓	PSNR ↑	SSIM ↑	Time ↓	Memory ↓
TV-FWI	19.94	0.7480	37.29	19.82	0.7707	37.84	18.09	0.6801	27.97	25.72	0.8875	21.67	18.51	0.7978	37.80	75.7
MS-FWI	20.29	0.7471	50.56	21.54	0.8580	50.57	20.68	0.7093	33.88	23.27	0.8248	32.86	22.96	0.9051	40.05	75.7
IFWI	25.08	0.9244	105.53	30.15	0.9666	55.14	22.45	0.9262	79.21	32.04	0.9855	58.04	27.95	0.9573	84.43	91.6
IG-FWI	16.62	0.8829	77.70	28.17	0.9568	51.28	28.76	0.9524	77.74	18.01	0.7629	53.35	21.77	0.8348	49.99	89.4
Ours	25.16	0.9292	3.36	24.01	0.9253	2.89	25.97	0.9545	6.25	26.39	0.9559	5.60	25.81	0.9418	20.83	25.9

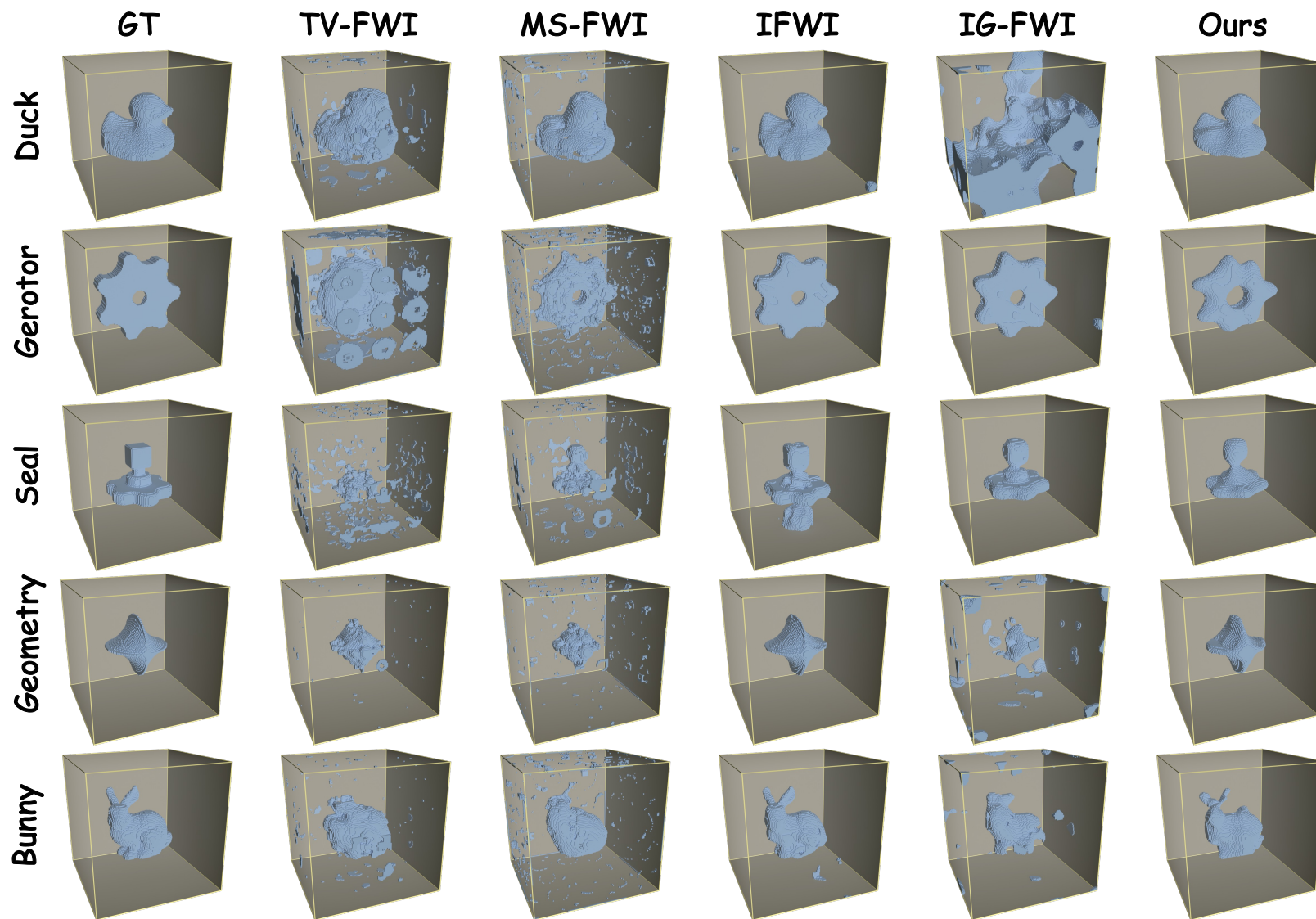
- Noisy condition

	10 dB		5 dB		0 dB	
	PSNR ↑	SSIM ↑	PSNR ↑	SSIM ↑	PSNR ↑	SSIM ↑
TV-FWI	20.59	0.7418	20.53	0.7372	20.43	0.7248
MS-FWI	21.85	0.7792	21.90	0.7682	21.71	0.7562
IFWI	22.93	0.8733	20.08	0.8255	19.29	0.7800
IG-FWI	19.52	0.8078	17.87	0.7758	16.06	0.7218
Ours	24.22	0.9302	23.84	0.9241	23.16	0.9196

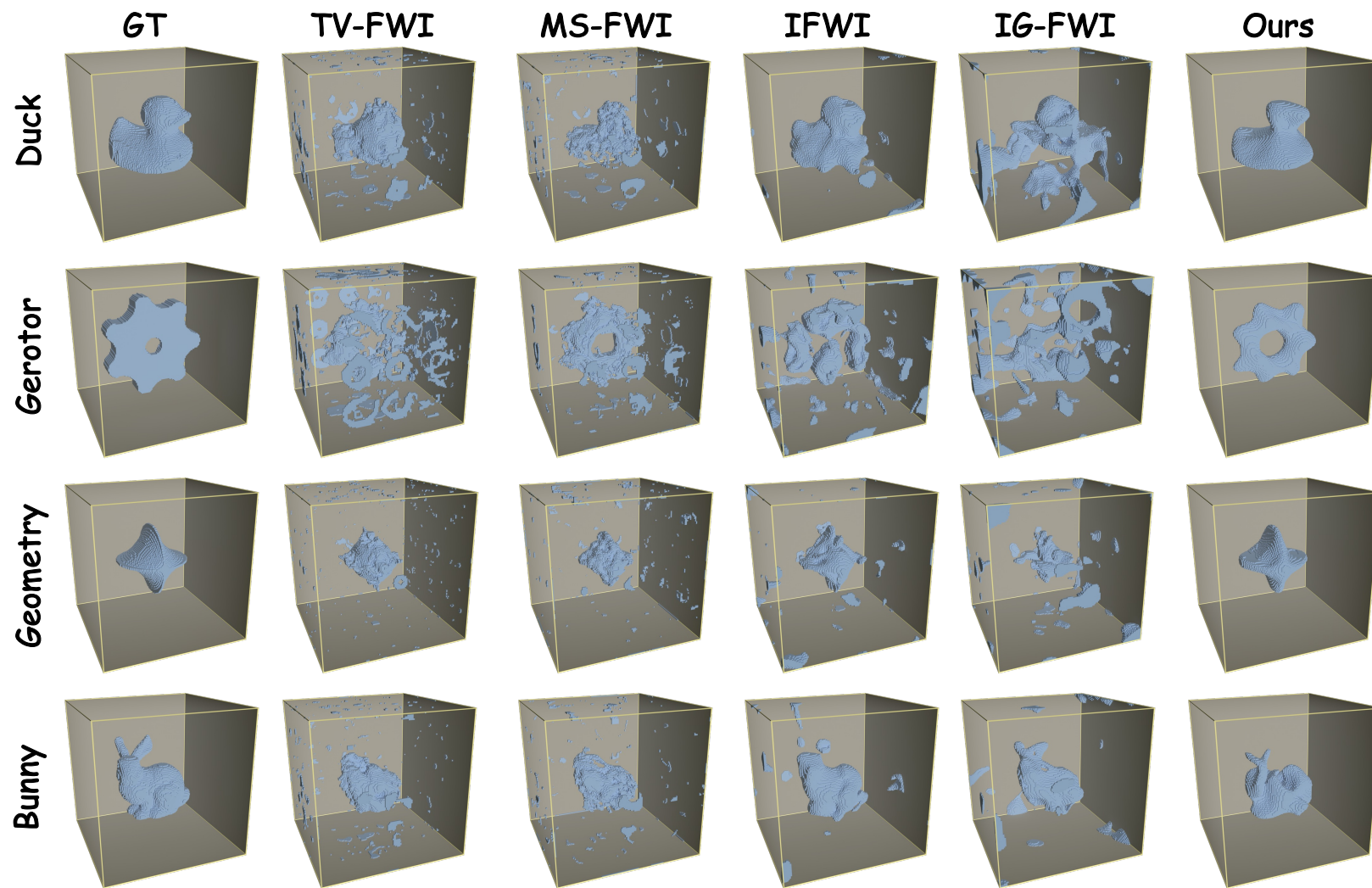
- Sparse acquisition condition

	2/3 measurements				2-view			
	PSNR ↑	SSIM ↑	Time ↓	Memory ↓	PSNR ↑	SSIM ↑	Time ↓	Memory ↓
TV-FWI	20.77	0.8011	9.82	41.0	21.37	0.8179	10.35	73.4
MS-FWI	21.93	0.8499	9.92	41.0	21.72	0.8472	10.38	73.4
IFWI	25.06	0.8888	10.49	59.8	22.37	0.8460	10.80	91.6
IG-FWI	17.73	0.7554	10.43	59.2	18.26	0.7718	10.51	89.4
Ours	25.15	0.9371	1.57	21.2	24.58	0.9309	1.66	21.9

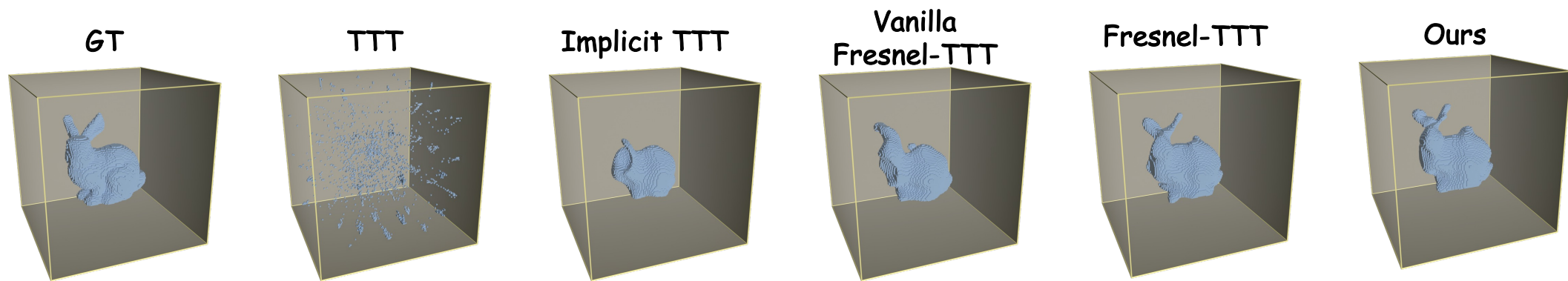
Qualitative results - Standard condition



Qualitative results - Noisy condition



Ablation study



	$PSNR \uparrow$	$SSIM \uparrow$	$Time \downarrow$
TTT	-	-	-
Implicit TTT	24.00	0.9315	16.08
vanilla Fresnel-TTT	24.64	0.9374	38.75
Fresnel-TTT	25.82	0.9447	40.92
Ours	25.81	0.9418	20.83

Future work

- Real-world applications
- Leveraging reflected-wave information

Thanks