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Gaeta–Lie Neural SDEs

Symmetry-Regularized Learning of Stochastic Dynamics

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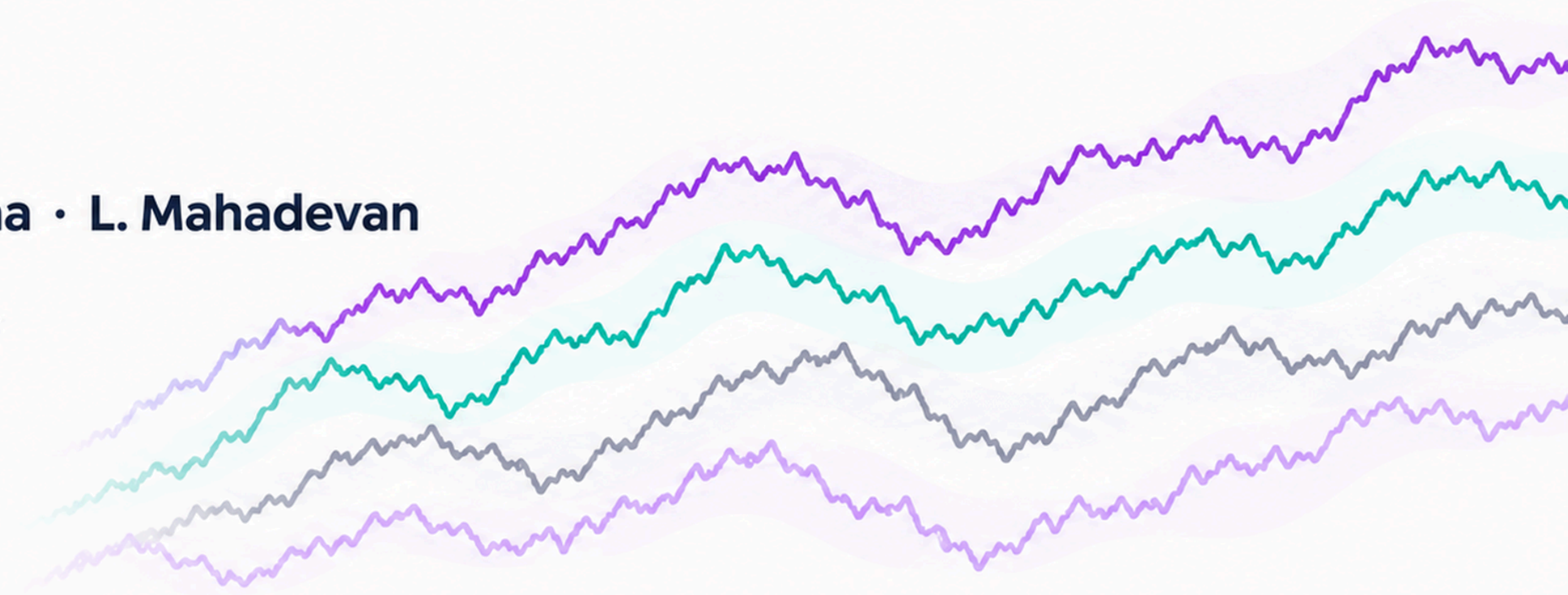


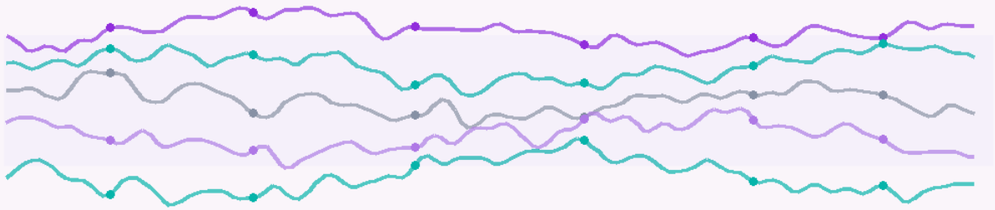
Matching trajectories is not enough

A good fit can still hide the wrong stochastic dynamics.

$$\mathrm{d}X_t = f(t, X_t)\,\mathrm{d}t + \Sigma(t, X_t)\,\mathrm{d}W_t$$

WHAT WE OBSERVE

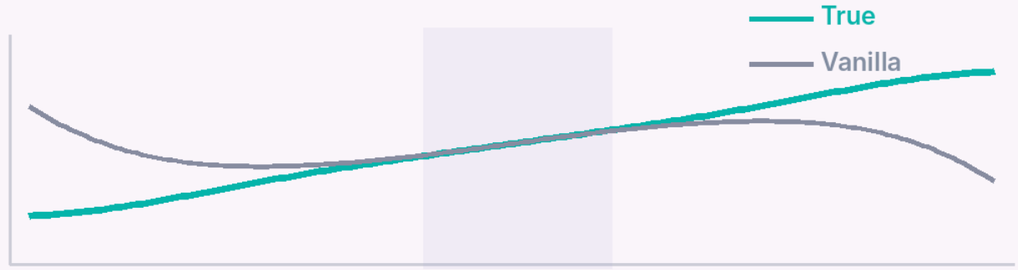
Limited, noisy trajectories



Observations only along sampled paths

WHAT WE NEED

Correct coefficients beyond the data



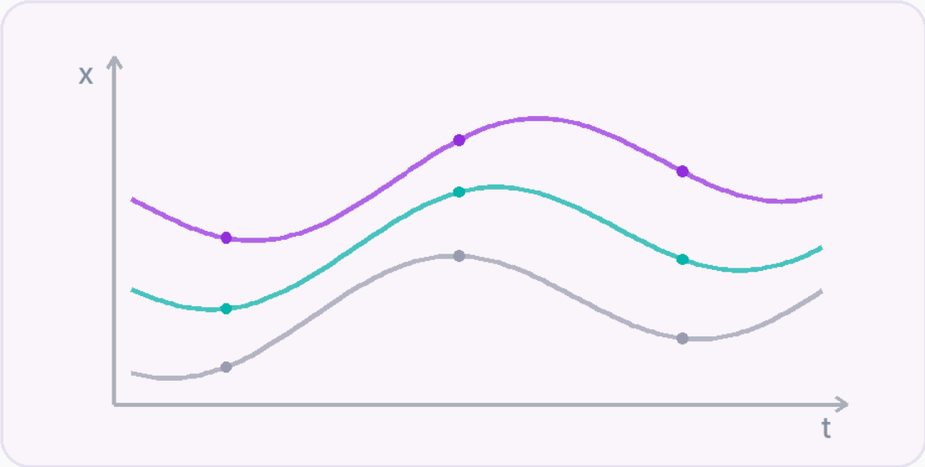
Observed region

Goal: recover the drift and diffusion, not just reproduce paths.

Symmetry reveals hidden structure

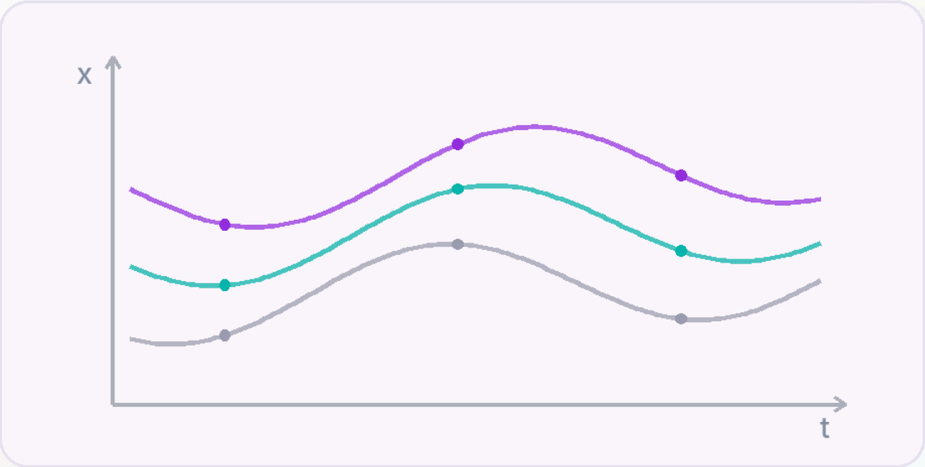
Change the coordinates. Preserve the stochastic law.

Original coordinates



Symmetry
transform

Transformed coordinates



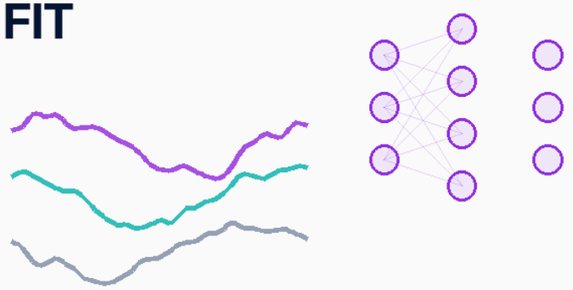
Approximate symmetry = small determining-equation residual $R_f \approx 0, \quad R_\Sigma \approx 0$

Gaeta–Lie: fit, discover, refine

Turn approximate symmetries into useful constraints.

01

FIT



Learn a neural SDE

02

DISCOVER

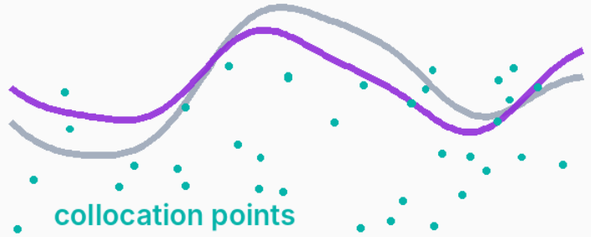
∂_t ∂_{x_i}

$x_i \partial_{x_i}$ $e^{\mu t} \partial_{x_i}$

Find symmetry generators

03

REFINE



Penalize symmetry defects

$$\mathcal{L}_{\text{refine}} = \mathcal{L}_{\text{data}} + \lambda_{\text{GL}} \mathcal{L}_{\text{det}}$$

Symmetry constraints can be checked at new time–state points.

Better recovery across dimensions

Average MSE reductions compared with Vanilla Neural SDEs

12 SYNTHETIC SYSTEMS

1D · 3D · 20D · 50D

−44%

DRIFT MSE

56%

−48%

DIFFUSION MSE

52%

DATA-SCARCE 50D

3 systems · 60 training trajectories

−62%

DRIFT MSE

38%

−37%

DIFFUSION MSE

63%

Uniform-box collocation also improved both coefficients in 50D.

Bars show remaining error relative to Vanilla = 100%.

Beyond coefficient recovery

The same geometric idea improves prediction and generation.

PhysioNet

ICU interpolation



15 / 15

settings improved

MuJoCo Hopper

Multistep forecasting



12 / 12

forecasts improved

MNIST

64D score-based generation



−12.7%

Fréchet distance

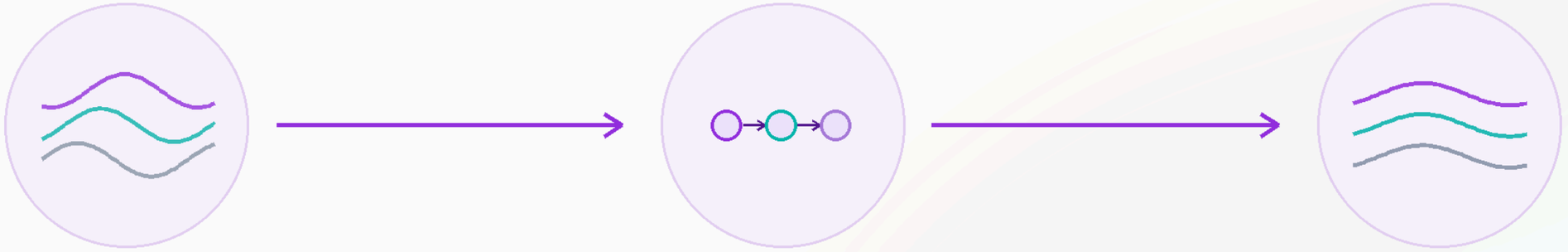
vs. equal-update continuation

Gaeta–Lie composes with task-specific neural SDE models.

Learn the symmetry.

Refine the dynamics.

Discovered approximate symmetries become a geometric prior.



FIT DISCOVER REFINE

No fixed drift or diffusion form · Benefits with limited data in our tests

Requires an informative initial surrogate and an adequate symmetry library.