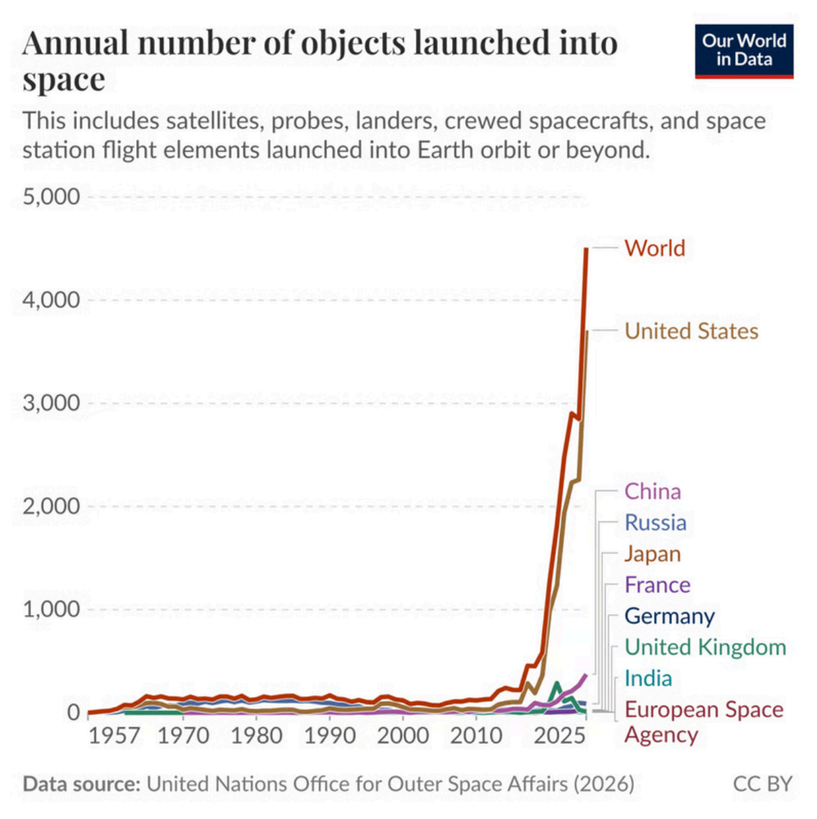
NEURAL INFORMATION PROCESSING SYSTEMS
NOVA SCHOOL OF SCIENCE & TECHNOLOGY
PRR Plano de Recuperação e Resiliência
REPÚBLICA PORTUGUESA
Financiado pela União Europeia NextGenerationEU
Trustworthy ML >_ NOVA Lab
NOVALINCS LABORATORY FOR COMPUTER SCIENCE AND INFORMATICS

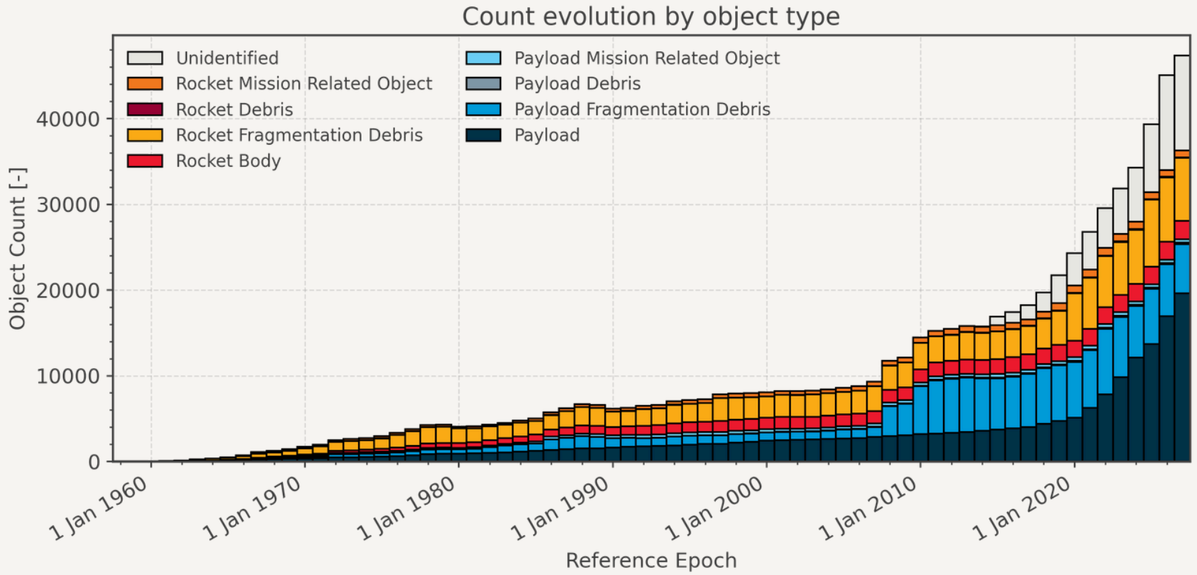
ASTRA: ADMM-Accelerated Topology Reconfiguration for Dynamic Satellite Constellations

João Norberto, *Ricardo N. Ferreira, Cláudia Soares*

Motivation

- Increasing number of satellites launched to space.
- Satellites provide day-to-day services.
- Characteristics of Low Earth Orbit (LEO) satellites.
- Use of Inter-Satellite Links to avoid relying on base stations to communicate.



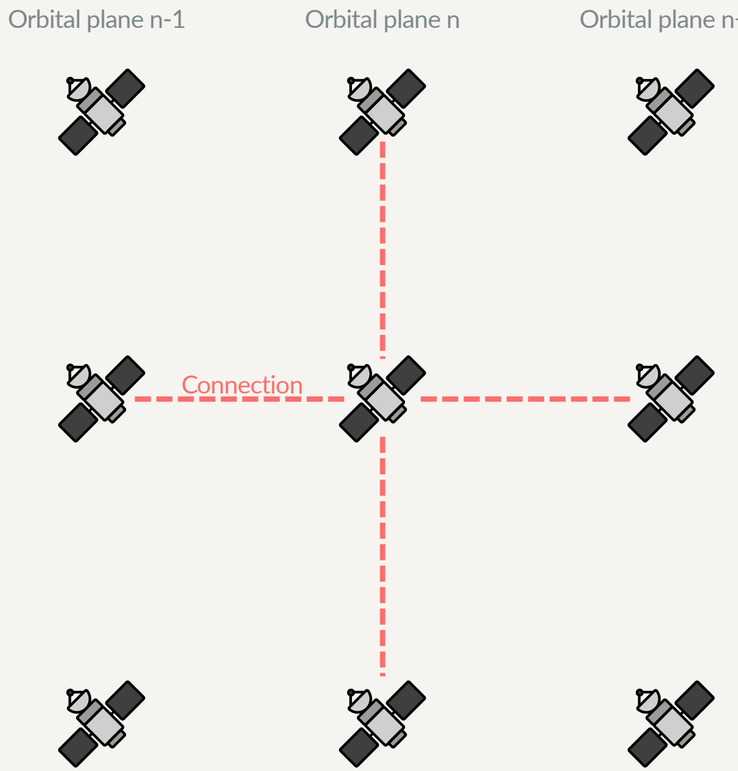


Source: European Space Agency - Space Environment report 2025 (https://sdup.esoc.esa.int/discosweb/statistics/static/allEvoTypeCnt.png)

Motivation

How to establish a topology:

- Static methods - Fix a connection pattern and apply it to all satellites.
  - Strengths: computationally cheap.
  - Weakness: do not consider the dynamics of space.



Example of connections established by **+Grid**.

Motivation

How to establish a topology:

- Static methods - Fix a connection pattern and apply it to all satellites.
  - Strengths: computationally cheap.
  - Weakness: do not consider the dynamics of space.
- Dynamic methods - Propose a topology given the environment characteristics.
  - Strengths: provide more reliable topologies.
  - Weakness: computationally expensive.

Trade-off: **Computational time** and **Adaptation to the dynamics of space**.

Motivation

How to establish a topology:

- Static methods - Fix a connection pattern and apply it to all satellites.
  - Strengths: computationally cheap.
  - Weakness: do not consider the dynamics of space.
- Dynamic methods - Propose a topology given the environment characteristics.
  - Strengths: provide more reliable topologies.
  - Weakness: computationally expensive.

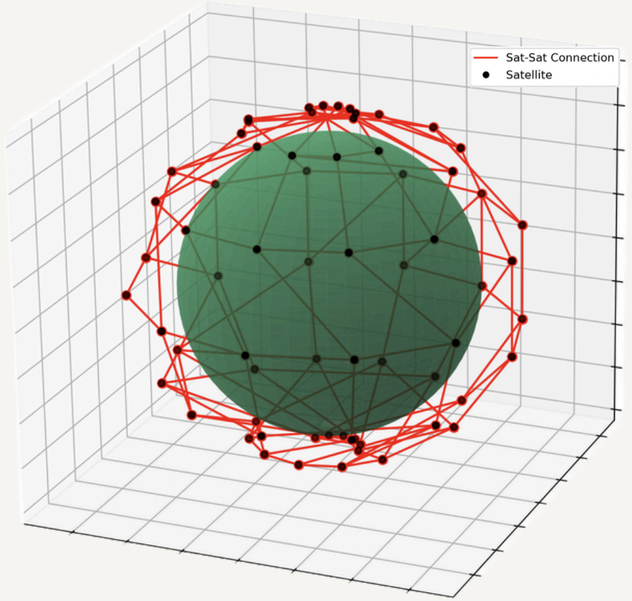
Trade-off: **Computational time** and **Adaptation to the dynamics of space**.

Problem: SOTA methods that tackle this problem either **assume idealized conditions** for satellite constellations or **only test their methods in these scenarios**.

Motivation - GUTO

Recent work modelled this problem as:

$$\begin{aligned} \underset{X \in \mathbb{R}^{n \times n}}{\text{minimize}} \quad & \frac{1}{2} \|(X - P) \odot U\|^2{}_F - \mathbf{Tr}(\Lambda \odot X) \\ \text{s.t.} \quad & X = X^T, \qquad X\mathbf{1} = \mathbf{0}, \qquad B^{\text{lower}} \leq X \leq B^{\text{upper}} \end{aligned}$$



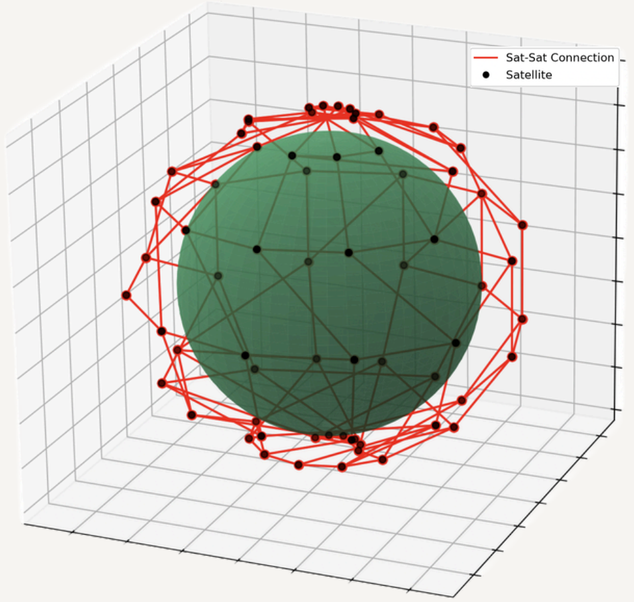
Example of generated topology

Motivation - GUTO

Recent work modelled this problem as:

$$\underset{X \in \mathbb{R}^{n \times n}}{\text{minimize}} \quad \frac{1}{2}\|(X - P) \odot U\|_F^2 - \mathbf{Tr}(\Lambda \odot X)$$
$$\text{s.t.} \quad X = X^T, \qquad X\mathbf{1} = \mathbf{0}, \qquad B^{\text{lower}} \leq X \leq B^{\text{upper}}$$

- ISL establishment via Frobenius norm minimization.
  - Feasible connection matrix - *P*
  - Utility matrix - *U*



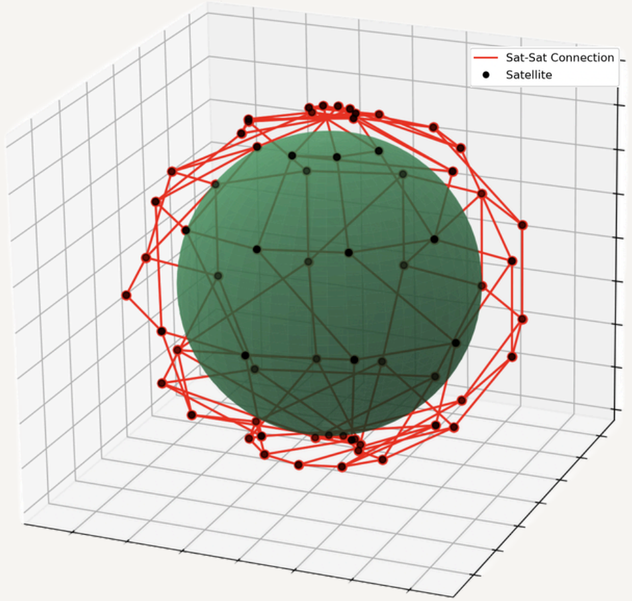
Example of generated topology

Motivation - GUTO

Recent work modelled this problem as:

$$\underset{X \in \mathbb{R}^{n \times n}}{\text{minimize}} \quad \frac{1}{2}\|(X - P) \odot U\|^2{}_F - \mathbf{Tr}(\Lambda \odot X)$$
$$\text{s.t.} \quad X = X^T, \qquad X\mathbf{1} = \mathbf{0}, \qquad B^{\text{lower}} \leq X \leq B^{\text{upper}}$$

- ISL establishment via Frobenius norm minimization.
  - Feasible connection matrix - *P*
  - Utility matrix - *U*
- Connectivity incentive via Trace maximization.
  - Weight matrix - Λ



Example of generated topology

Motivation - GUTO

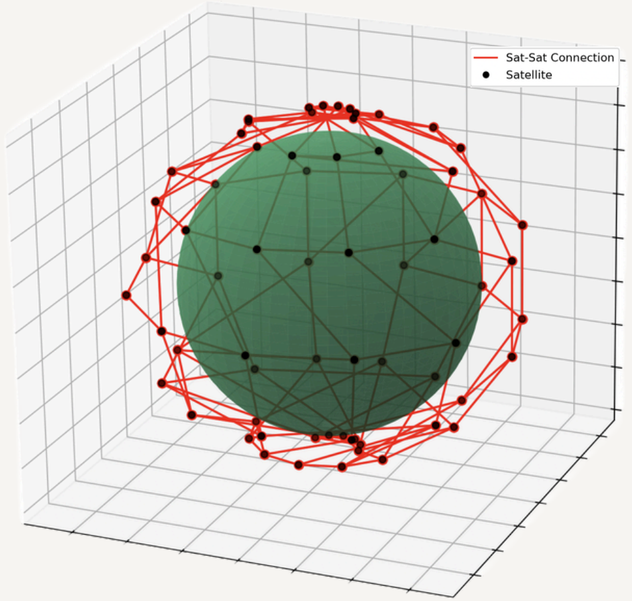
Recent work modelled this problem as:

$$\begin{aligned} \underset{X \in \mathbb{R}^{n \times n}}{\text{minimize}} \quad & \frac{1}{2}\|(X - P) \odot U\|_F^2 - \mathbf{Tr}(\Lambda \odot X) \\ \text{s.t.} \quad & X = X^T, \qquad X\mathbf{1} = \mathbf{0}, \qquad B^{\text{lower}} \leq X \leq B^{\text{upper}} \end{aligned}$$

- ISL establishment via Frobenius norm minimization.
  - Feasible connection matrix - *P*
  - Utility matrix - *U*
- Connectivity incentive via Trace maximization.
  - Weight matrix - Λ

Limitations:

- Relies on generic solvers.
- No theoretical backing.
- Only applied to ideal constellation assumptions.
- Limited Online Learning analysis.



Example of generated topology

Main Contributions

We make that formulation substantially more practical and theoretically sharper by **introducing ASTRA**. Specifically, we:

- Derive a problem-structured ADMM solver.
- Accelerate the online methods.
- Establish stronger regret guarantees for this topology objective.
- Connect the theory to the implemented inexact solver.
- Validate beyond idealized constellation assumptions.

Main Contributions - ASTRA Formulation

We make that formulation substantially more practical and theoretically sharper by **introducing ASTRA**. Specifically, we:

- Derive a problem-structured ADMM solver.
  - Introduce our ADMM-based solver:

Algorithm : ASTRA

Require: initialize X^1, Z^1, V^1, ρ, and $k = 1$.

while some stopping criteria are not met **do**
  $X^{k+1} = \text{ProjectedGradient}(X^k)$
  $Z^{k+1} = \min\left(\max\left(X^{k+1} + V^k, B_{ij}^{\text{lower}}\right), B_{ij}^{\text{upper}}\right)$
  $V^{k+1} = V^k + \left(X^{k+1} - Z^{k+1}\right)$
  $k = k + 1$
end while

Main Contributions - ASTRA Formulation

We make that formulation substantially more practical and theoretically sharper by **introducing ASTRA**. Specifically, we:

- Derive a problem-structured ADMM solver: Introduced the **offline ASTRA algorithm**.
- Accelerate the online methods.
  - We derive ADMM-based subroutines for two online algorithms:

Algorithm : ASTRA projection for OGD and OCG

Require: Initialize X^1, Z^1, V^1, ρ, receive $\overline{X}_t$ as input and $k = 1$.

while some stopping criteria are not met **do**

$W^k = \frac{\overline{X}_t + \rho(Z^k - V^k)}{1+\rho}$ or $(Z^k - V^k) - \frac{\nabla F_t(X_t)}{\rho}$

$X^{k+1} = \Pi_K(W^k)$

$Z^{k+1} = \min\left(\max(X^{k+1} + V^k, B_{ij}^{\text{lower}}), B_{ij}^{\text{upper}}\right)$

$V^{k+1} = V^k + (X^{k+1} - Z^{k+1})$

$k = k + 1$

end while

return X^k

Main Contributions - Theoretical Guarantees

We make that formulation substantially more practical and theoretically sharper by **introducing ASTRA**. Specifically, we:

- Derive a problem-structured ADMM solver: Introduced the **offline ASTRA algorithm**.
- Accelerate the online methods: Derived **ADMM-based** subroutines for **OGD** and **OCG**.
- Establish stronger regret guarantees for this topology objective.
  - We show that the loss function $f(X) = \frac{1}{2}\|(X - P) \odot U\|_F^2 - \mathbf{Tr}(\Lambda \odot X)$ is **strongly convex** for **entry-wise non-zero utility matrices.**
  - Hence, **OGD** obtains a **logarithmic static regret bound**:

$$\mathrm{Regret}_T \leq \frac{L^2}{2\mu}\left(1 + \log\left(T\right)\right)$$

- Connect the theory to the implemented inexact solver.
  - We analyze **dynamic regret** under **inexact ADMM inner loops**. By carefully selecting the learning rate of **OGD**, we obtain the standard dynamic regret bound:

$$\mathrm{DynamicRegret}_T = O\left(\sqrt{TP(u_1, ..., u_n)}\right)$$

Main Contributions - Results

We make that formulation substantially more practical and theoretically sharper by **introducing ASTRA**. Specifically, we:

- Derive a problem-structured ADMM solver: Introduced the **offline ASTRA algorithm**.
- Accelerate the online methods: Derived **ADMM-based** subroutines for **OGD** and **OCG**.
- Establish stronger regret guarantees for this topology objective: **Logarithmic** static regret for **OGD**.
- Connect the theory to the implemented inexact solver: **Optimal** dynamic regret for **OGD with a tailored learning rate**.
- Validate beyond idealized constellation assumptions.
  - ASTRA **matches** GUTO's performance in both settings. These methods **outperform** the SOTA in **real-world Starlink data**.

Metrics	E ↑	A-Deg ↑	CC ↓	A-SP ↓	Time (s) ↓
+Grid	130	3.94	1	4.036	**0.001**
×Grid	124	3.76	1	4.978	**0.001**
DoTD	131	3.97	1	4.412	0.023
Motif	**132**	**4.00**	1	4.292	1.326
GUTO	128	3.88	1	**3.767**	0.091
ASTRA	128	3.88	1	**3.767**	0.071

Graph metrics on an Iridium-like constellation

		Metrics			
Method:	FOV	E↑	A-Deg↑	CC↓	A-SP* ↓
+Grid	—	$2869.14_{\pm 14.123}$	$3.608_{\pm 0.017}$	$9.038_{\pm 2.019}$	✗
×Grid	—	$2468.77_{\pm 19.325}$	$3.205_{\pm 0.026}$	$3.346_{\pm 1.314}$	✗
DoTD	—	$\mathbf{3183.16_{\pm 0.586}}$	$\mathbf{3.998_{\pm 0.001}}$	$4.211_{\pm 1.436}$	✗
GUTO	H	$2900.95_{\pm 13.033}$	$3.650_{\pm 0.015}$	$\mathbf{1.233_{\pm 0.491}}$	$15.742_{\pm 0.388}$
ASTRA	H	$2900.24_{\pm 13.058}$	$3.649_{\pm 0.015}$	$1.264_{\pm 0.533}$	$15.760_{\pm 0.381}$
GUTO	H+C	$2797.16_{\pm 23.664}$	$3.544_{\pm 0.025}$	$1.642_{\pm 0.878}$	$\mathbf{11.771_{\pm 0.226}}$
ASTRA	H+C	$2795.63_{\pm 23.506}$	$3.542_{\pm 0.025}$	$1.646_{\pm 0.878}$	$11.817_{\pm 0.224}$

Graph metrics on real-world Starlink data

  - **ASTRA** is **faster** than **GUTO** but slower than static methods, demonstrating the **trade-off** between **computational time** and **adaptation to the dynamics of space**.

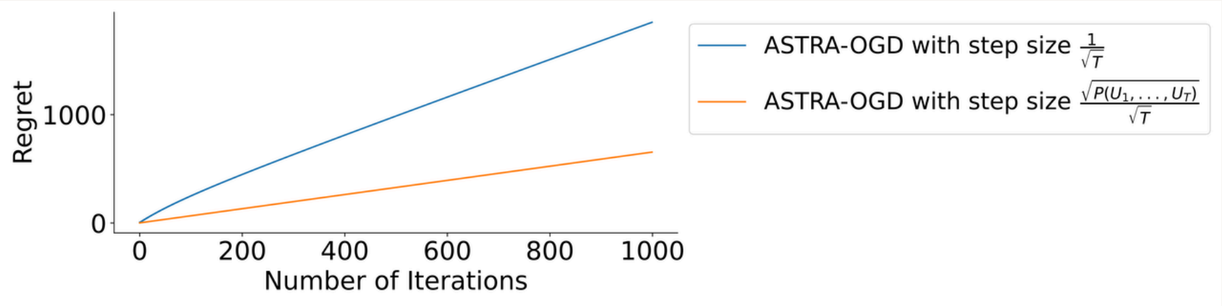
Method:	+Grid	×Grid	DoTD	GUTO	ASTRA
Time (s)	$0.021_{\pm 0.001}$	$0.026_{\pm 0.001}$	$3262.927_{\pm 48.097}$	$665.027_{\pm 2.093}$	$567.949_{\pm 5.615}$

Time comparison to propose a single Starlink topology

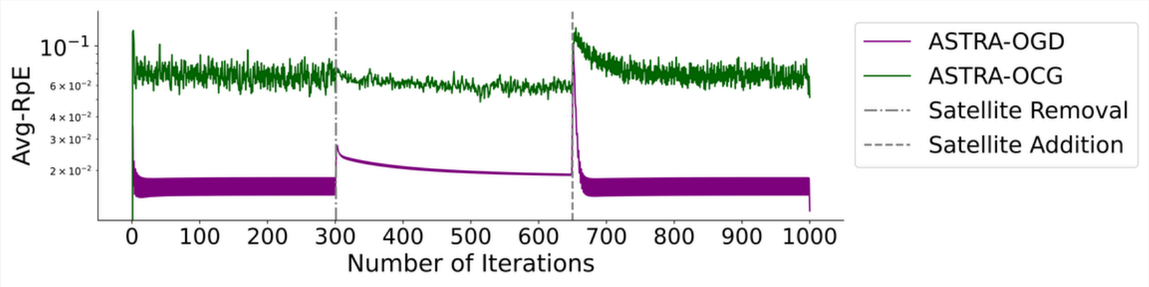
Main Contributions - Results

We make that formulation substantially more practical and theoretically sharper by **introducing ASTRA**. Specifically, we:

- Derive a problem-structured ADMM solver: Introduced the **offline ASTRA algorithm**.
- Accelerate the online methods: Derived **ADMM-based** subroutines for **OGD** and **OCG**.
- Establish stronger regret guarantees for this topology objective: **Logarithmic** static regret for **OGD**.
- Connect the theory to the implemented inexact solver: **Optimal** dynamic regret for **OGD with a tailored learning rate**.
- Validate beyond idealized constellation assumptions.
  - **ASTRA-OGD** with a tailored learning rate, **considering the path length** obtains **smaller regret values**.



  - **ASTRA-OGD** is capable of adapting to satellite **removal and addition**, while **ASTRA-OCG struggles with satellite addition**.



Conclusion

With this paper we showed:

- ASTRA **matches** the performance of GUTO, while being **computationally cheaper**.
- ASTRA obtains **comparable** results with the SOTA in **idealized constellations**.
- **SOTA** methods **fail under non-idealized conditions**, contrary to ASTRA and GUTO.
- Provide stronger **theoretical guarantees** for **ASTRA-OGD**.
- **ASTRA-OGD** is a **competitive** online solution.

Thank You

João Norberto - jd.norberto@campus.fct.unl.pt
Ricardo N. Ferreira - rjn.ferreira@campus.fct.unl.pt
Cláudia Soares - claudia.soares@fct.unl.pt